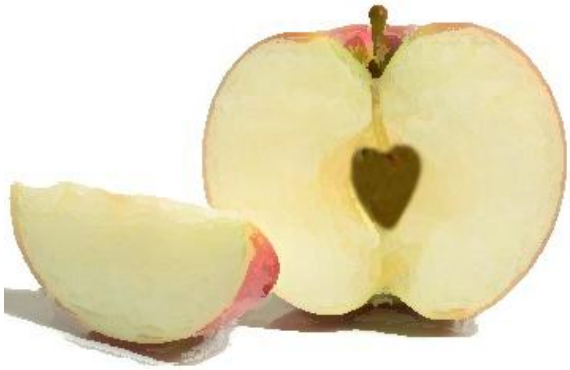

Getting to the Core

Algebra II

A9 – Functions – Unit of Study

Updated on May 3, 2013

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Unit A9 – Functions
Table of Contents

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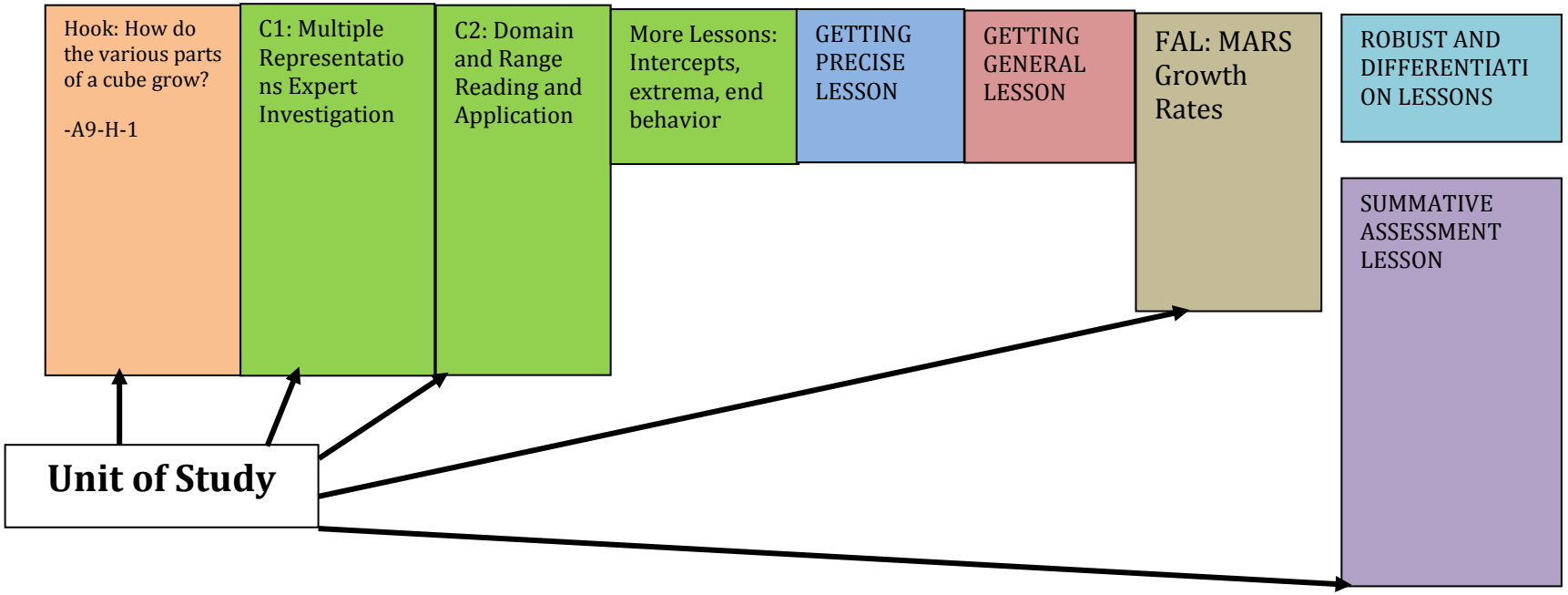
Unit Title:	A9 – Functions	
Grade Level/Course:	Algebra 2 / CC3	**Time Frame:** Entire Unit might be about 4 weeks. This segment requires 7-8 days.
Big Idea (Enduring Understandings):	Functions describe quantitative change. Various families of functions have different qualities and key features, making them useful as models in different situations.	
Essential Questions:	What are functions? How can they be represented? How are they useful?	

Instructional Activities: Activities/Tasks

Units have many types of lessons that have different **purposes**



Designing for Opportunities for Standards for Mathematical Practice happen at the Unit Level
Presented by Bill McCallum, Ph.D., Algebra Forum 2012

<table>
<tr><td>21st Century Skills:</td><td colspan="2">Learning and Innovation:
☒ Critical Thinking & Problem Solving ☒ Communication & Collaboration ☒ Creativity & Innovation
Information, Media and Technology:
☐ Online Tools ☐ Software ☐ Hardware</td></tr>
<tr><td>Essential Academic Language:</td><td>Tier II:

Functions
Linear
Quadratic
Exponential
Cubic
Logarithmic
Rational
Growth functions
Rate of change
y-intercepts
domain
range</td><td>Tier III:

Multiple representations
Match
Corresponding</td></tr>
<tr><td colspan="2">What pre-assessment will be given?

Formative Assessment
• Preparing the Learners
• Function notations
• Representations of Linear, Quadratic, and Cubic Functions</td><td>How will pre-assessment guide instruction?

Formative Assessment
• Prepare students to be in different expert groups to either strengthen their prior knowledge and clear misconceptions or to deepening the prior knowledge in different contexts.</td></tr>
<tr><td colspan="2">Standards</td><td>Assessment of Standards (include formative and summative)</td></tr>
<tr><td colspan="2">Common Core Learning Standards Taught and Assessed (include one or more standards for one or more of the areas below. Please write out the complete text for the standard(s) you include.)</td><td>What assessment(s) will be utilized for this unit? (include the types of both formative assessments (F) that will be used throughout the unit to inform your instruction and the summative assessments (S) that will demonstrate student mastery of the standards.)</td><td>What does the assessment tell us?</td></tr>
</table>

Common Core Mathematics Content Standard(s): **N.RN: The Real Number System** **Extend the properties of exponents to rational exponents.** **N.RN.1 –** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define$5^{1/3}$ to be the cube root of 5 because we want $\left(5^{1/3}\right)^3 = 5^{(1/3)^3}$ to hold, so $(5^{(1/3)^3}$ must equal 5. **N.RN.2 –** Rewrite expressions involving radicals and rational exponents using the properties of exponents. **A.SSE** **Interpret the structure of expressions** **A-SSE.1 –** Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P **A-REI: Reasoning with Equations and Inequalities** **Represent and solve equations and inequalities graphically.** **A-REI.11 –** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$and $y = g(x)$interects are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g. using technology to graph the functions, make	**Formative Assessments:** Preparing the Learner Lesson • Assessing students' prior knowledge to determine appropriate lessons to either strengthen and deepen the knowledge or clear off misconceptions. **Summative Assessments:** • Assessing students' just learned knowledge in a summative assessment that asks students to apply prior knowledge and skills presented in this unit to a modeling problem.	• To meet the diverse needs of students within Rigor: Procedural, Conceptual, or Application • Have students apply their knowledge in modeling problems

tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. **F.IF.4** **Interpret functions that arise in applications in terms of the context.** **F-IF.4 –** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: *intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ** **F.IF.7 –**Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* a. b. Graph square root, cube root, and piecewise-defined functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **F.BF.1 –** Write a function that describes a relationship between two quantities.* a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard functions types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to decaying exponential, and relate these functions to the model. **F.BF.3 –** Identifying the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x+k) for specific values of k (both positive		

and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* **F-LE: Functions-Linear, Quadratic, and Exponential Models*** **Construct and compare linear, quadratics, and exponential models and solve problems.** **F.LE.3 –**Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.		
Opportunities for listening, speaking, reading, writing, and thinking (*Cite Literacy Standards (as applicable):* **Bundled Language Standards:** 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. 6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases, including those that signal contrast, addition, and other logical relationships (e.g., however, although, nevertheless, similarly, moreover, in addition) **Bundled Speaking and Listening Standards:** 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics and texts, building on others' ideas and expressing their own clearly. a. Come to discussions prepared having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion. b. Follow agreed-upon rules for discussions and carry out assigned roles. c. Pose and respond to specific questions by making comments that contribute to the discussion and elaborate on the remarks of others.		

d. Review the key ideas expressed and draw conclusions in light of information and knowledge gained from the discussions. 4. Report on a topic or text, or present an opinion, sequencing ideas logically and using appropriate facts and relevant, descriptive details to support main ideas or themes; speak clearly at an understandable pace.		

Standards of Mathematical Practice:	(*Check all that apply*) ☒ 1. Make sense of problems and persevere in solving them. ☐ 2. Reason abstractly and quantitatively. ☒ 3. Construct viable arguments and critique the reasoning of others. ☒ 4. Model with mathematics. ☐ 5. Use appropriate tools strategically. ☐ 6. Attend to precision. ☐ 7. Look for and make use of structure. ☐ 8. Look for and express regularity in repeated reasoning.	**Opportunities for Observable Data** *(How will students demonstrate these Mathematical Practices?)* Mathematical Practices are being demonstrated via: • Collaborative Group Discussion • Reasoning and Explanation via communication on daily work • Reasoning on formative and summative assessments
Resources/ Materials:	**Text(s) Titles:** **Mathematical Tools:** **Media/Technology:** **Supplementary Materials:**	
Interdisciplinary Connections:	**Cite several interdisciplinary or cross-content connections made in this unit of study (i.e. literature, science, social studies, art, etc.)**	

Differentiated Instruction:	**Based on desired student outcomes, what instructional variation will be used to address the needs of English Learners by language proficiency level?** • Use of sentence frames (appropriate for language level) to facilitate academic language and conversations • Use of visual organizers (thinking maps) to assist processing mathematical ideas • Explicitly teaching key academic vocabulary • Use of mathematical tools (graphing calculator) to facilitate conceptual understanding • Flexible grouping to support language acquisition and target instruction • Use of collaboration to promote socio-cultural learning	**Based on desired student outcomes, what instructional variation will be used to address the needs of students with special needs, including gifted and talented?** **Special Needs-** • Use of sentence frames to help with speaking, reading, and writing • Use of visual organizers (thinking maps) to organize and evaluate material • Explicitly teaching key academic vocabulary **GATE-** •

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SAUSD Common Core Lesson Planner Mathematics Teacher:_____

Unit: A9 **Lesson:** **A9-H-1**	**Grade Level/Course:** CC3/Algebra 2	**Duration: <u>One Period of One (50-minute)</u>** **Date:**
Common Core and Content Standards	**Analyze functions using different representations** F-IF 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **Interpret functions that arise in applications in terms of the context** F-IF 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	
Materials/ Resources/ Lesson Preparation	Student Intro Activity Page	
Objectives	**Content:** Students will calculate various features of a cube including surface area and volume, recognizing the different features and representations of linear, quadratic and cubic functions.	**Language:** • Students will use academic vocabulary to explain patterns and how they formulated equations to represent each function. • Opportunities for listening, speaking, reading, writing and thinking via Bundled Language Standards: 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. 6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases, including those that signal contrast, addition, and other logical relationships (e.g., however, although, nevertheless, similarly, moreover, in addition)
Depth of Knowledge Level	☒**Level 1: Recall** ☒**Level 2: Skill/Concept** ☐ **Level 3: Strategic Thinking** ☐ **Level 4: Extended Thinking**	
Standards for Mathematical Practice	☒ **1. Make sense of problems and persevere in solving them.** ☒ **2. Reason abstractly and quantitatively.** ☒ **3. Construct viable arguments and critique the reasoning of others.** ☐ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☐ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	
ca bul ary (Ti **TEAC HER SIMPL E EXPL**	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**

		Function Exponential Linear Quadratic Cubic Rate of changes y-intercept	Multiple Representations Matches/Corresponds
	STUDENTS FIGURE OUT THE MEANING		
Pre-teaching Considerations		(Content) Have students seen/worked with function notation? (Content) Have students seen the graphs of linear, quadratic, and cubic functions?	

Lesson Delivery

Instructional Methods

Check method(s) used in the lesson:

☐ **Modeling** ☐ **Guided Practice** ☒ **Collaboration**

☐ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**

Prior Knowledge, Context, and Motivation:

Prior Knowledge: Students have significant experience with linear and quadratic functions, which they will use to fill out the table and make predictions. This lesson focuses attention on the idea that there are all types of functions that can be represented in several different ways.

Context: This lesson will introduce students to the multiple representations for different functions.

Motivation: Students will see functions in a very tangible and relevant way spiking curiosity and motivation to apply what they already know to solve problems in different contexts.

Lesson Continuum

Body of the Lesson:
Activities/ Questioning/ Tasks/ Strategies/ Technology/ Engagement

Lesson Overview

Day 1 of 1:

Teacher: Introduce problem to students and have students form pairs to work on activity.

15 Minutes:

- **Independent Group Work**
- **Mathematical Practice Being Monitored:** Make sense of problems and persevere in solving them.

In pairs, students will work with one another to figure out what the problem is asking and how they can calculate the number of straws and sheets of paper to fill in the table.

Teacher: Walk around room monitoring and guiding students in the right direction by asking leading questions

15 Minutes:

Differentiated Instruction:

English Learners:
Ask students to rephrase the task in their own words.
Teacher supplies sentence frames for the students.

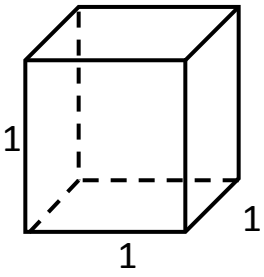
Students Who Need Additional Support:

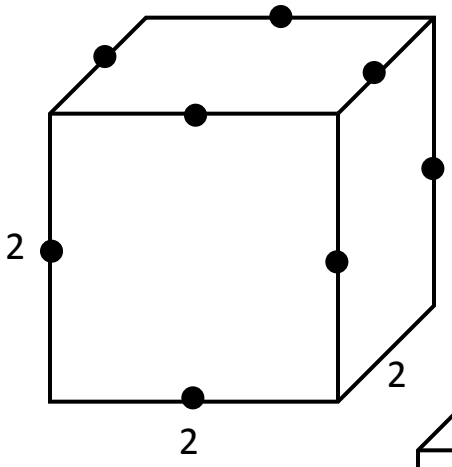
- Ask students to rephrase the task in their own words.
- Teacher supplies

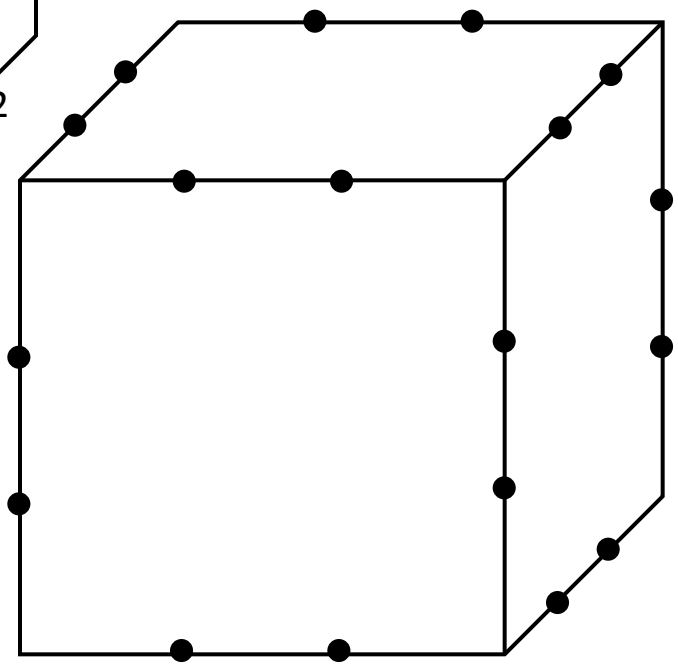
	• **Mathematical Practice Being Monitored:** Reason abstractly and quantitatively **Students:** Work in pairs to make sense of the relationship in each column, figuring out what is happening every time x increases by one. Ie: the number of straws increases by 12 each time or 12 is multiplied by x Recognizing these patterns, students can then develop equations to represent the relationship and use it to calculate the value of any x for all three columns and answer the corresponding questions. **15 Minutes:** • **Mathematical Practice Being Monitored:** Construct viable arguments and critique the reasoning of others. **Teacher:** Bring the class together for a discussion of the activity. Goal is to help students see that there are different ways to represent functions – we can create tables, use equations, use words, use pictures, and use graphs to represent the same function. Walk students through discussion questions on handout. Discuss the differences between the different columns of the table. Ask leading questions to get students curious and eager to share their findings and reasoning. **Suggested Set of Guided Inquiry/Questions:** • How did you calculate the number of straws when you first started? • Did that change as you continued and noticed a pattern? • What type of function is it when it increases by the same amount every time? • How is this different from the second column where you calculated the sheets of paper to cover each side? • What equation did you come up with? What is the power of x? • What type of function is it when there is an x^2 ? • What was the last column calculating? • What equation did you come up with? • Have you seen the graph of x^3 ? Most may not be familiar with cubic functions. If time permits, introduce them to cubic functions and their graphs. Summarize findings and what a great job students did formulating all types of representations for 3 main functions. So what are the different ways we can represent functions? Remember this, we will be using this as we work through this unit for all types of functions.	sentence frames for the students • For any of the partner work, some students with special needs may need to be paired with a student strong in math. **Accelerated Learners:** The accelerated learner on a team can act as the Task Manager by listening for statements and reasons. He can ask questions such as, "Explain how/why you know that", or "Can you explain this in a different way?"
Lesson Reflection		
Teacher Reflection Evidenced by Student Learning/ Outcomes		

Cube Growth **Name _____Period _____**

Jason is creating cubes with different side lengths out of straws and construction paper. He needs your help to figure out how many straws (each straw is 1 ft) and sheets of paper (1ft by 1ft) he needs. Help Jason calculate how many materials he should purchase by completing the following table.





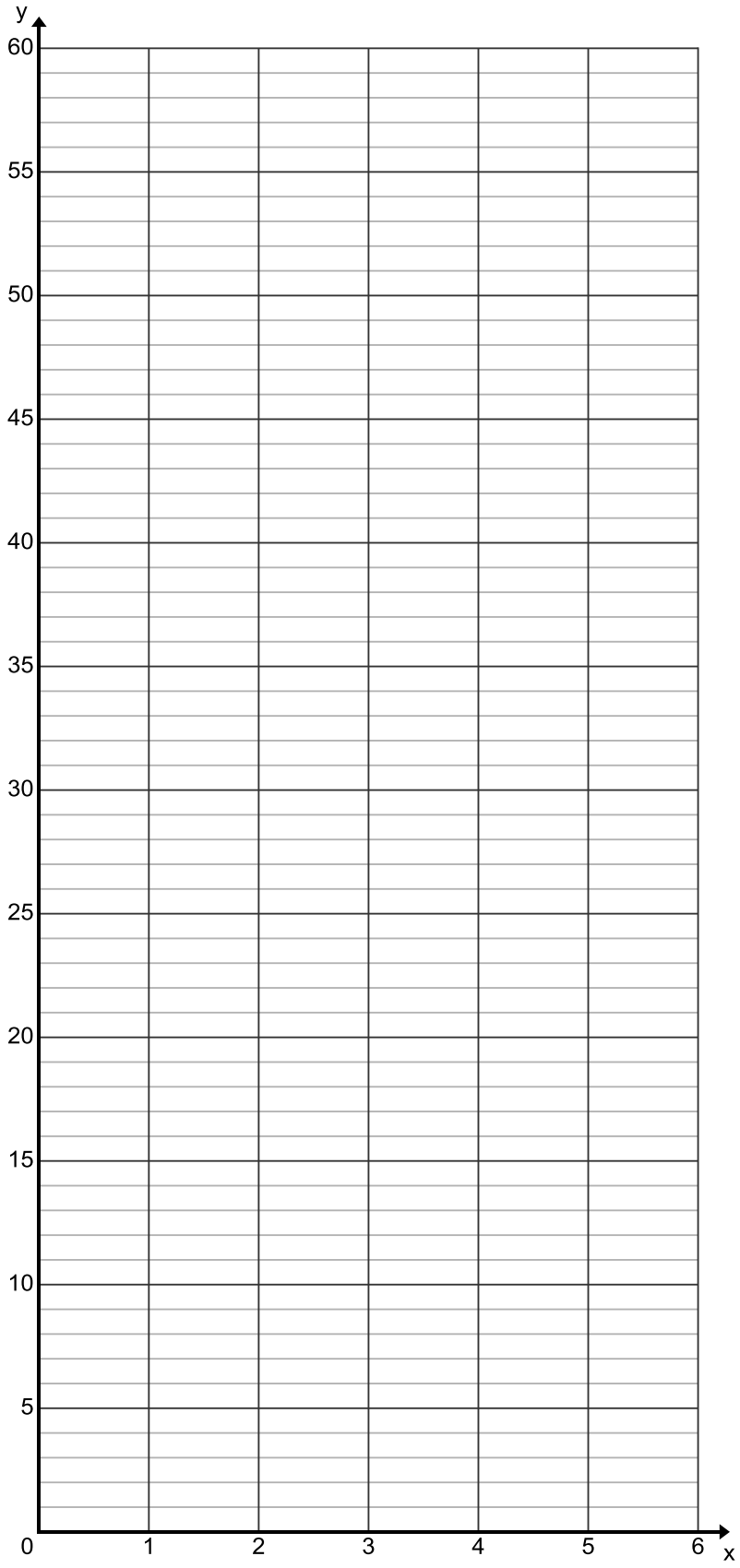


Start by counting the straws and papers for a cube with edges of 1 foot, then count for the 2nd and 3rd cube.

Next, use the patterns or your own drawings to fill in the rows for 0, 4 and 5.

x (in feet)	How many straws Jason would need?	How many sheets of construction paper Jason would need to cover the sides?	What is the volume of the cube that Jason makes?
0			
1			
2			
3			
4			
5			

Note: the volume of a cube is the number of 1 ft cubes that will fit inside it.



Use the data from your table and three different colors to make three graphs in this coordinate plane.

One graph will be for the number of straws.

A second will be for the number of 1ft by 1ft squares of paper.

The third will be for the volume.

1. How many straws do you think Jason will need if x = 6?

    a. How do you know? Explain the pattern.

    b. Is there an equation you can create to represent the number of straws Jason will need given any x value? Write the equation.

    c. What does this graph look like? What type of function is this?

    d. How many straws will Jason need if x =10? Show your work.

2. How many sheets of construction paper do you think Jason will need if x = 6?

    a. How do you know? Explain the pattern.

    b. Is there an equation you can create to represent the sheets of paper Jason will need given any x value? Write the equation.

    c. What does this graph look like? What type of function is this?

    d. How many sheets of paper will Jason need if x = 10? Show your work.

3. What would be the volume if Jason builds a cube with x = 6?

    a. How do you know? Explain the pattern.

    b. Is there an equation you can create to represent the volume for any x value? Write the equation.

    c. What does this graph look like? What type of function is this?

    d. What will be the volume if x = 10? Show your work.

SAUSD Common Core Lesson Planner Mathematics Teacher:_____

<table>
<tr><td>Unit: A9
Lesson:
A9-PTL-A</td><td>Grade Level/Course:
Alg2/CCC3</td><td colspan="2">Duration: <u>Day 1 of One Period (50 Minutes)</u>
Date:</td></tr>
<tr><td>Common Core and Content Standards</td><td colspan="3">F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
Note: the objective for this lesson is to provide the skill set necessary for the work of the next lesson and to use strategies to help support the collaborative group work for the later lessons.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Formative Assessment: Functions Unit Pre-Assessment
Functions Unit Skill Review Handout</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:

Students are to demonstrate and strengthen the fluency of substitution and function notations, graphing, and writing linear, quadratic, and radical equations.</td><td>Language:

Students are to read, think, discuss, share, and analyze problems written for fluency of skills.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☒Level 1: Recall ☒Level 2: Skill/Concept
☐ Level 3: Strategic Thinking ☐ Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="3">☐ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☒ 3. Construct viable arguments and critique the reasoning of others.
☐ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☒ 6. Attend to precision.
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td colspan="2">Common Core Instructional Shifts in Mathematics</td><td colspan="2">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="2">Academic Vocabulary (Tier II & Tier III)
PROVIDES TEACHER SIMPLE EXPLANATION</td><td>KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td>f(x) – read as “f of x”
substitution</td><td></td></tr>
</table>

	STUDENTS FIGURE OUT THE MEANING		
Pre-teaching Considerations		N/A	

Lesson Delivery

Instructional Methods	**Check method(s) used in the lesson:** ☒ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☐ **Independent Practice** ☐ **Guided Inquiry** ☒ **Reflection**
	Prior Knowledge, Context, and Motivation: **Substitution** **Graphing given a set of values** **Simple evaluation and calculation with integers**

Lesson Continuum

Body of the Lesson: Activities/ Questioning/ Tasks/ Strategies/ Technology/ Engagement	**Lesson Overview** **All explorations are collaborative in nature:** **Day 1 of 1** **20 Minutes** **Functions Unit Pre-Assessment Handout** **Teacher:** Have students work on the Pre-Assessment Handout individually. Students are given 20 minutes to work on this assessment to recall the necessary skills for the next lessons. The objective for this assessment is to inform both students and teachers of what skills they need to strengthen on. Pre-Assessments handout will inform teacher of what skills students are in need for clarification and they are **Part 1:** substitution and interpretation of function notation, **Part 2:** graphing, and **Part 3:** writing equations of linear, quadratic, and radical functions. **Students:** individually work on the assessment. **5-10 Minutes** **Teacher's objective:** Identify students' needs and group them in the expert groups of skills: **Part 1:** substitution and notation, **Part 2:** graphing or **Part 3:** writing equations. **Student:** if miss at 50% of an identified part, is to go to the specified group to sharpen his/her skills with the similar group for clarification. **20-25 Minutes** **Teacher:** have students go back to their original group to work on the **Functions Unit Skill Review** handout. Please provide the following structure to support group work by asking leading questions. 1. How does your neighbor/team member(s) get this answer? 2. What makes a particular problem hard? How did you go about solving it? What are some of the resources that you could use to solve that problem? 3. Any trend/pattern you've found in each section of the handout? How are they the same? How are they different? How did you find the difference and how does that help you in completing the work?	**Differentiated Instruction:** **English Learners:** **Students Who Need Additional Support:** **Accelerated Learners:**

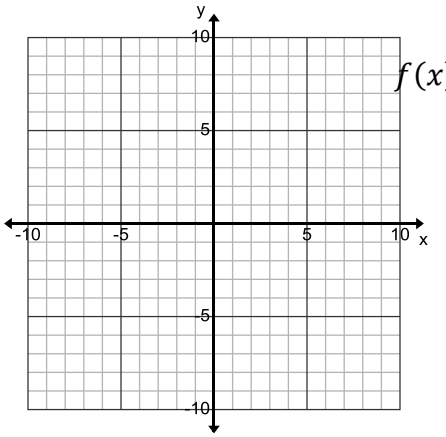
	Students: Are to work collaboratively in pairs or group on the problems. **Teacher:** Once students are done with the handout, have each group make a poster and perform a Gallery Walk on this handout. **Students:** Are to answer as a team on the following questions with examples and explanation why. 1. The most challenging problem on this handout. Explain the challenges and how the group came about solving that problem. 2. The least challenging problem and why. 3. An interesting problem and what makes it interesting and how the team went about solving them. **Gallery Walk Structure:** • Each group will display their poster • Each group selects a group member to be the docent to answer questions or provide clarifications/explanations • The other group members examines, explores, reviews the other groups' posters • There will be time for each group to re-assemble and discuss the information shared in the groups' posters • Please remind gallery walk norms and be respectful of the work and information shared. **Teacher:** Please assign tasks for the rest of the team to pay attention to while performing the walk. Assign a purpose for each observant. Prompts: Student 1: What is a method or problem that is found common across teams? Student 2: What is a unique method that is very different from the rest of the team? Student 3: Pick one method/poster that gives a different answer from yours. Evaluate that method in comparison to yours. **Students:** Are to be back in their group and share their findings after the Gallery Walk.	
Lesson Reflection		
Teacher Reflection Evidenced by Student Learning/ Outcomes		

Functions Unit Pre-Assessment **Name** _____

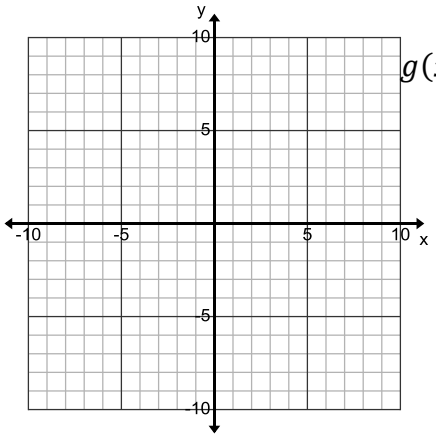
Part 1: For each of the functions given below, find the indicated values.

$f(x) = 3x$ $f(5) =$ $f(-2) =$	$g(x) = x^2$ $g(4) =$ $g(-3) =$	$h(x) = x^3$ $h(3) =$ $h(-5) =$
$f(x) = \sqrt{x}$ $f(16) =$ $f(100) =$	$g(x) = 3^x$ $g(4) =$ $g(-3) =$	$h(x) = x^3 - 3x^2$ $h(4) =$ $h(-4) =$

Part 2: Draw the graphs of the following functions.

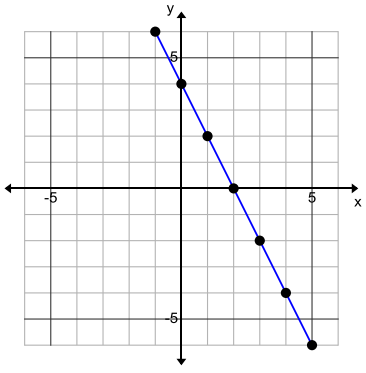
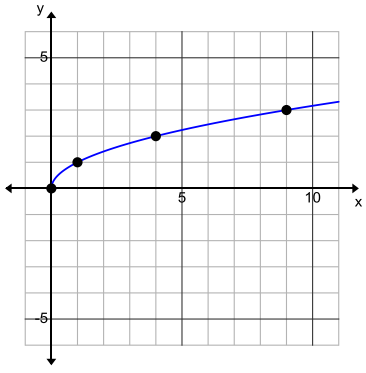
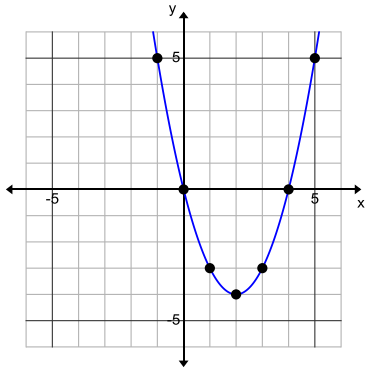


$f(x) = 3x - 4$



$g(x) = x^2 - 6$

Part 3: Identify the function in each graph.

		
$f(x) =$	$g(x) =$	$h(x) =$

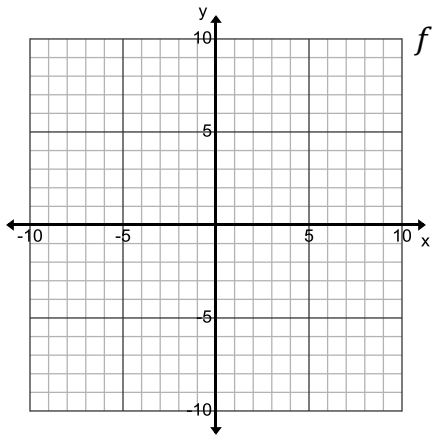
Functions Unit Skill Review **Name** _____

Note: $f(x) = 5x$ is read "f of x equals five x" and $f(4)$ is read "f of 4" and is the y-value or function value at x=4.

For each of the functions given below, find the indicated values.

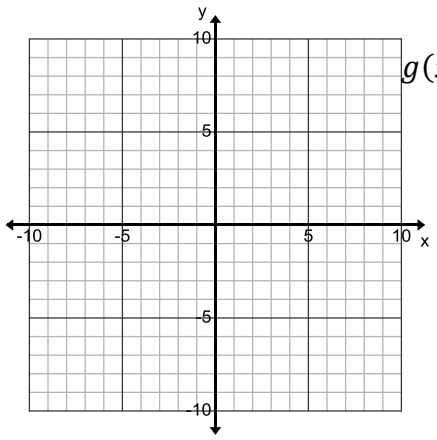
$f(x) = -x + 2$ $f(5) =$ $f(-2) =$	$g(x) = 3x^2$ $g(4) =$ $g(-3) =$	$h(x) = x^3 - 4$ $h(3) =$ $h(-5) =$
$f(x) = \sqrt{x}$ $f(9) =$ $f(81) =$	$g(x) = 4^x$ $g(2) =$ $g(-3) =$	$h(x) = x^2 + 4x + 3$ $h(2) =$ $h(-3) =$

Draw the graphs of the following functions by calculating function values for each input.



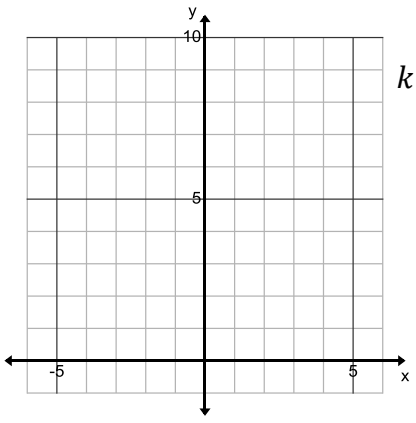
$f(x) = \frac{1}{2}x - 2$

x	y
-2	
0	
2	
4	
6	



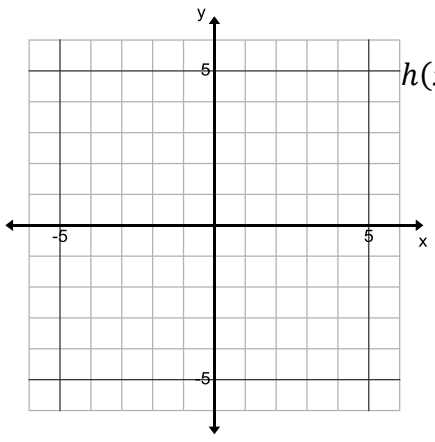
$g(x) = x^2 - 4$

x	y
-2	
-1	
0	
1	
2	



$k(x) = 3^x$

x	y
-2	
-1	
0	
1	
2	



$h(x) = \sqrt{x+5}$

x	y
-5	
-4	
-1	
4	
11	

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SAUSD Common Core Lesson Planner Mathematics Teacher:_____

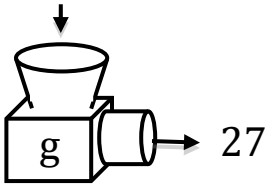
Unit: A9 **Lesson: A9-1-1**	**Grade Level/Course:** CC3/Algebra 2	**Duration: Two periods of 50 mins Lesson** **Date:**
Common Core and Content Standards	**Analyze functions using different representations** F-IF 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **Interpret functions that arise in applications in terms of the context** F-IF 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **Construct and compare linear, quadratic, and exponential models and solve problems** F-LE 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. F-LE 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
Materials/ Resources/ Lesson Preparation	Function Exploration Pages (25) Presentation guidelines Poster paper or presentation page for Elmo use. Summary of Key Features page (at least one for each team).	
Objectives	**Content:** Students will match different representations of a function to each other. Students will identify key features of five different families of functions and recognize those features in the various representations of the functions.	**Language:** Students will use academic vocabulary to explain matches between different representations of the same function. Students will listen to and critique other students' explanations regarding function representations and key features.
Depth of Knowledge Level	☐ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☐ **Level 3: Strategic Thinking** ☐ **Level 4: Extended Thinking**	
Standards for Mathematical Practice	☐ **1. Make sense of problems and persevere in solving them.** ☒ **2. Reason abstractly and quantitatively.** ☒ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☐ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	
ca bul ary (Ti TEACHER SIMPLE EXPL	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**

		Function Exponential Linear Quadratic Cubic Radical Square Root Starting Point Intercept Rate of Change	Multiple Representations Matches/Corresponds
	STUDENTS FIGURE OUT THE MEANING		
Pre-teaching Considerations		(Content) Have students seen/worked with function notation? (Content) Have students seen the graphs of linear, quadratic, cubic, exponential and radical functions? (Pedagogical) Have students moved about the classroom interacting with each other and math content? (Pedagogical) Have students presented conclusions and conjectures to the class before? **Refer to Preparing the Learner Lesson A before teaching this lesson.**	

Lesson Delivery

Instructional Methods	**Check method(s) used in the lesson:** ☐ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☐ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**	
	Prior Knowledge, Context, and Motivation: **Prior Knowledge:** Students have significant experience with linear, quadratic and exponential functions and some familiarity with cubing and square roots. This lesson draws on those experiences to develop the broad concept of functions in general, and growth functions in particular, as an idea that encompasses all five of these specific types. It also focuses attention on the idea that all of these functions can be represented in a table, as an equation/rule, as a graph, in a series of drawings, and in a verbal situation. **Context:** One of the five representations for each function is a simplified (not elaborate) context for the growth pattern. The other four are abstract and must be matched to each other and the context. **Motivation:** Competence with the five functions and the interdependence of needing to find the four other representations increases the motivation of students and helps them overcome the challenges of developing a better conceptual understanding of functions.	
Body of the Lesson: Activities/ Questioning/ Tasks/ Strategies/ Technology/ Engagement	**<u>Lesson Overview</u>** Day 1: **10 Minutes:** • **Mathematical Practice Being Monitored:** Reason abstractly and quantitatively. Run a Math Talk on the idea of "function." Start by drawing a function machine on the board, showing an input of 3 and an output of 27. Also, write the symbols: g(3) =27.	**Differentiated Instruction:** **English Learners:** Strategic pairing may be required for some students When students are in teams try using the Round Robin strategy so that all students are given the opportunity

Lesson Continuum



$$g(3) = 27$$

Teacher: Prompt students with a series of questions to think about what a function is, what types of functions they have learned about, how we read g(3)=27 and what function might take an input of 3 and turn it into an output of 27. You may use a strategy such as Think-Pair-Share, or have students discuss in small teams.

Suggested Set of Guided Inquiry/Questions:

- Complete this statement: A function is like a _____ because . . .
- What types of functions do you know about?
- How do you read the symbols g(3) = 27?
- What does g(3)=27 mean?
- If g(3)=27, what do you think g(5) will be?
- How many different functions can you think of in 1 minute that will turn the number 3 into the number 27?
- What are some of the ways a function can be described or shown?

Students: Think independently and then discuss in pairs or teams the questions posed about functions. Participate in a class discussion.

Note: All of the following are examples of functions that have g(3)=27.

$g(x) = 10x - 3$	$g(x) = 3^x$	$g(x) = 3x^2$	$g(x) = x^2 + 6x$
$g(x) = x^2 + 3x + 9$	$g(x) = 9x$	$g(x) = x^3$	$g(x) = 8x + 3$

30 Minutes:

- **Mathematical Practice Being Monitored:** Construct viable arguments and critique the reasoning of others.

The entire class will participate in a matching activity to put together five different representations each for five different growth functions.

Step 1: The 25 different pages are distributed to students in the class. Most students will have a page of their own, but some students will be paired with a partner and a page. (If there are 35 students in the class, 15 individual students and 10 pairs will receive pages.)

Step 2: List on the board the five different representations that are being used today: Table, Graph, Equation, Verbal Story, Pictures. Designate five different locations in the room – one for each of these types of representations. Tell everyone go to the location that matches what they have. Once there, have everyone look at and listen to the description of everyone else's function. Notice what is the same and what is different.

Step 3: Let everyone know that each page from the Table area has numbers that match one page from the Graph area, one page from the Equation area, one page from the Verbal Story area, and one page from the Pictures area. Their next job is to find the four other pages that match their function and sit down with them.

to speak and contribute.

Students Who Need Additional Support:

- Strategic pairing may be required for some students
- When students are in teams try using the Round Robin strategy so that all students are given the opportunity to speak and contribute.

- Provide students with special needs their team and job assignment the day prior to the activity so that they can prepare for the group work with their case carrier (not necessary for all, but for the ones who struggle with communication, math, group work, being put on the spot…this alleviates some of the stress and they'll feel better about participating. It will also benefit their group).

Accelerated Learners:

- The accelerated learner on the team can act as the Task Manager by listening for statements and reasons. He can ask questions such as, "Explain how/why you know that", or "Can you explain this in a different way?"

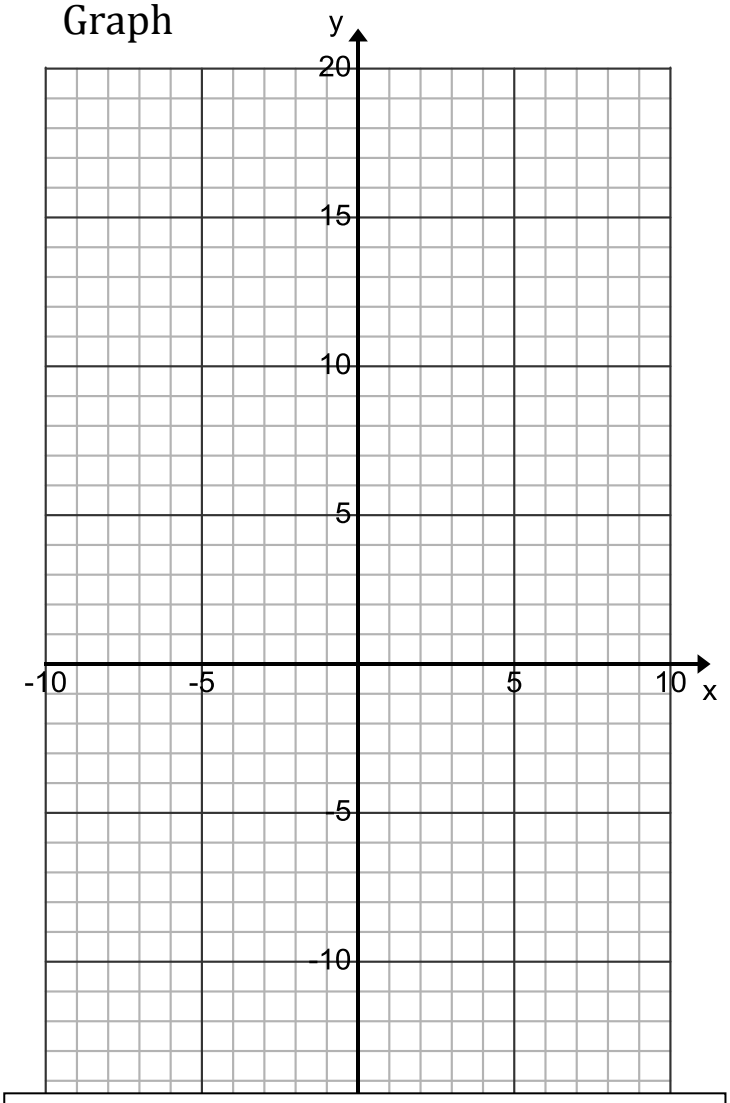
	Step 4: Ensure that all five pages match at each new team and have them share information to complete each other's pages. **10 Minutes:** • **Mathematical Practice Being Monitored:** Model with mathematics. Students begin recording observations, comments and questions about their function at the bottom of their page in preparation of presenting some key features and ideas about it to the class tomorrow. Students must each comment on the representation they began with, but must make connections to the other representations of the same function. Teacher: Encourage students to notice important features about the function, such as starting point (y-intercept) and the rate at which the function is changing. These key features appear in different ways in each of the different representations. **Day 2:** **20 Minutes:** • **Mathematical Practice Being Monitored:** Model with mathematics. Construct viable arguments and critique the reasoning of others. Reason abstractly and quantitatively. Reassemble students into their function teams. Give each team guidelines for their presentation and either a blank function page for the ELMO or an enlarged poster of a blank function page. Give each team a copy of the Summary of Key Features page to write about the connections between the representations. Some possible items for the presentation guidelines: • Everyone must speak and describe a connection between two representations. • Some parts of the page must be left blank with the class asked to complete them. The presentation must be interactive with the class. • Academic language must be used and used correctly. 25 **Minutes**: • **Mathematical Practice Being Monitored:** Model with mathematics. Construct viable arguments and critique the reasoning of others. Reason abstractly and quantitatively. Each of the five teams presents information from their function. Everyone in the class receives four blank function pages to use to record information from the other four presentations. **10 Minutes:** Quick-write: Explain what a function is. What are some of the similarities and differences between the five different function types you have studied?	
Lesson Reflection		
Teacher Reflection Evidenced by Student Learning/ Outcomes		

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) = 2^x$$

Situation

Drawing

Record your observations and questions here:

_____

_____

_____

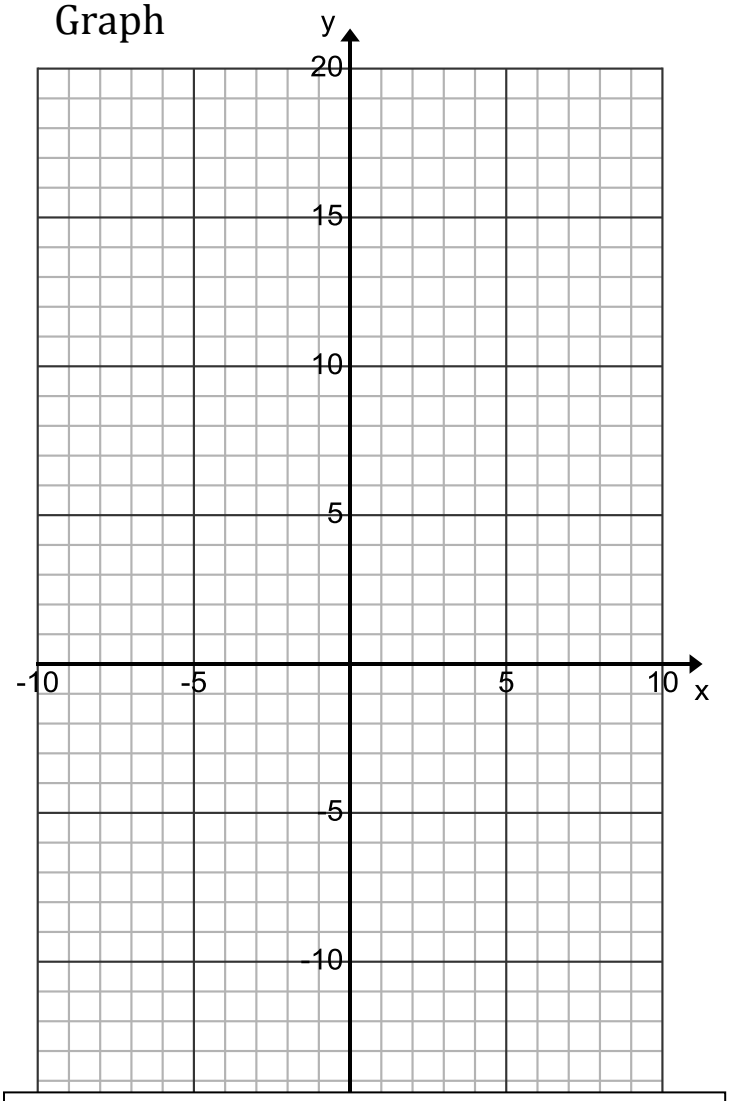
_____

_____

Data Table

x	f(x)
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512

Graph



Model Equation

$f(x) =$

Situation

Drawing

Record your observations and questions here:

_____

_____

_____

_____

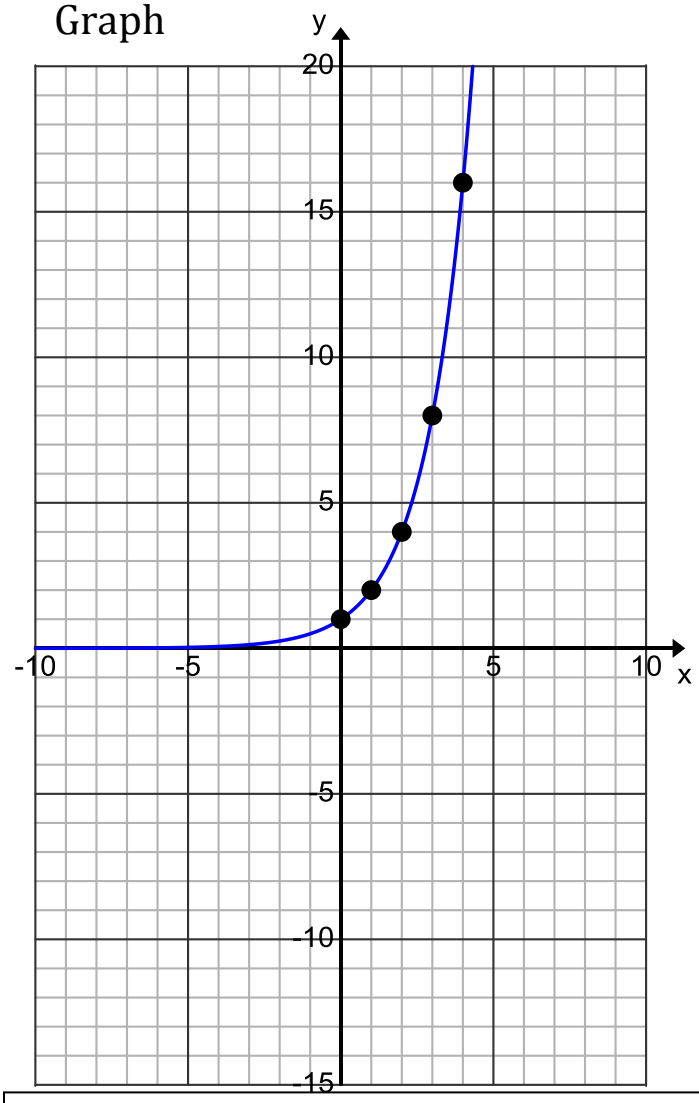
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$f(x) =$

Situation

Drawing

Record your observations and questions here:

_____

_____

_____

_____

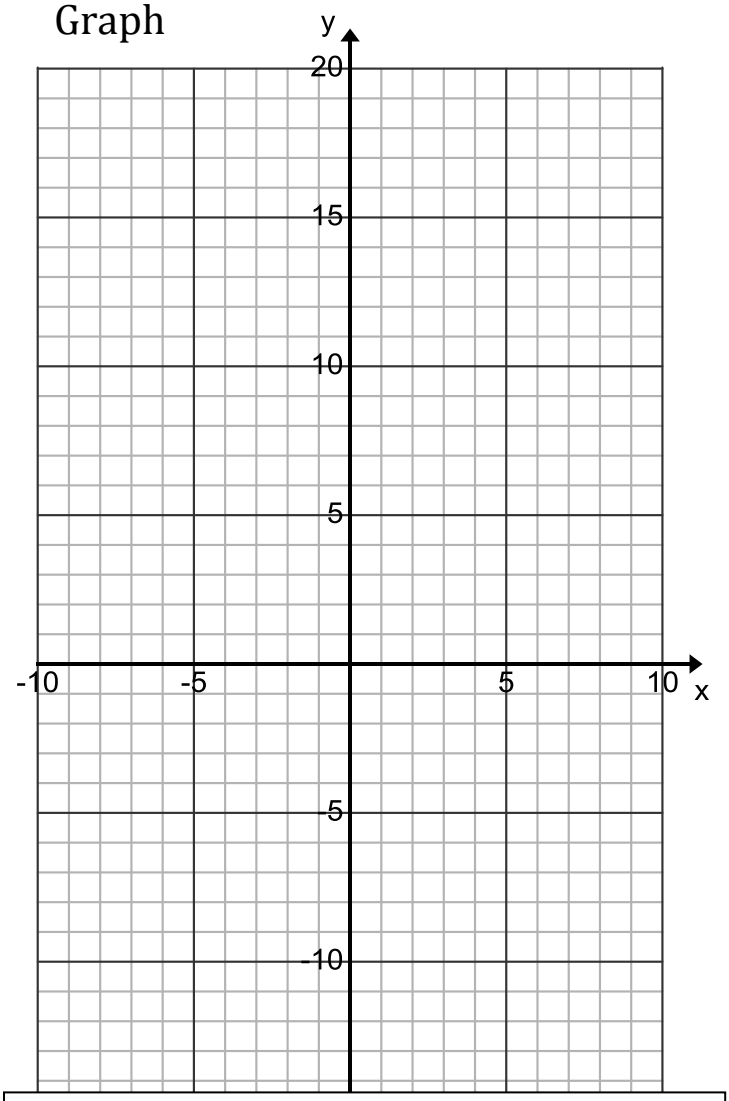
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$f(x) =$

Situation

Ezra starts with just one block, but doubles his amount on the first day. He continues to double his amount every day.

Drawing

Record your observations and questions here:

_____

_____

_____

_____

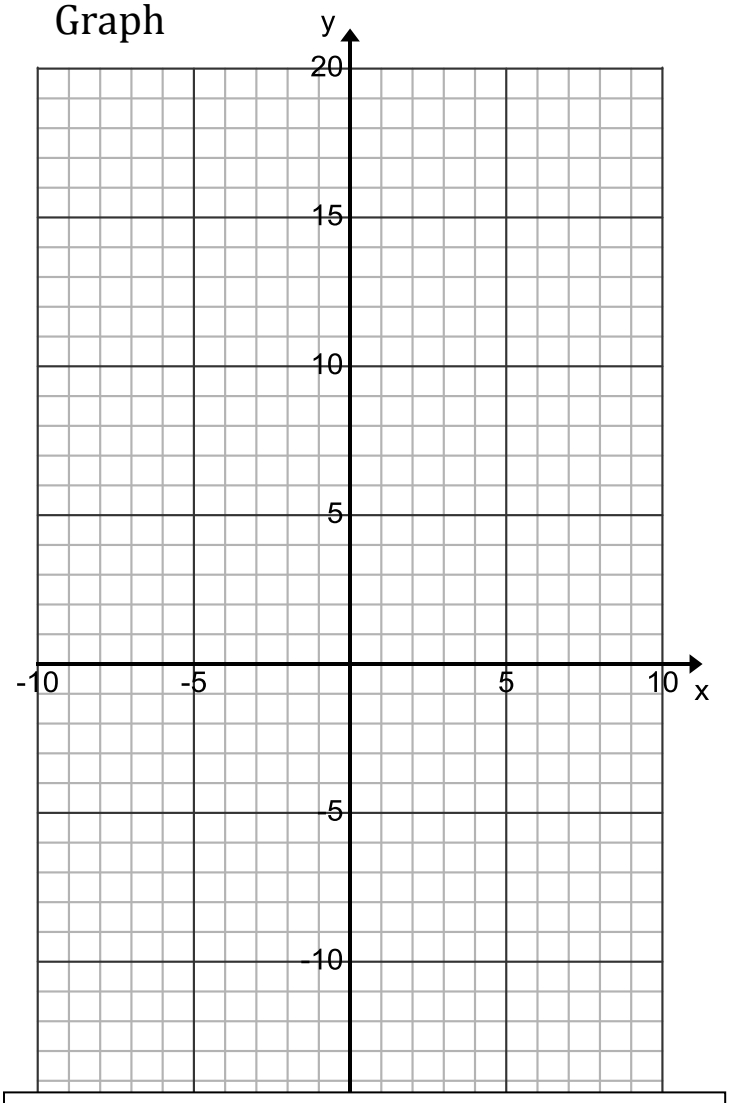
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph

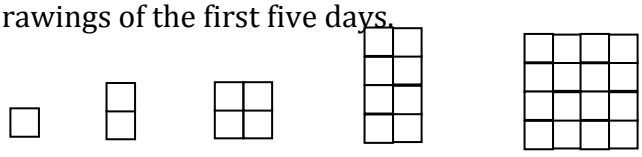


Model Equation

$f(x) =$

Situation

Drawings of the first five days.



Record your observations and questions here:

_____

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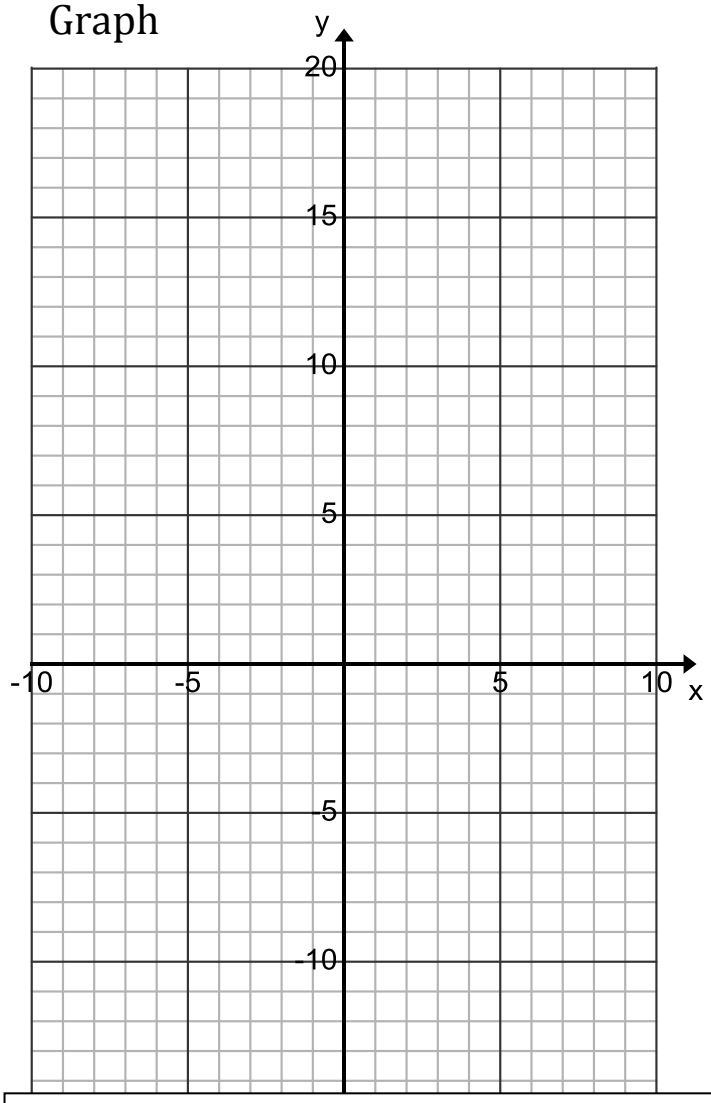
_____

_____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) = \sqrt{x}$$

Situation

Drawing

Record your observations and questions here:

_____

_____

_____

_____

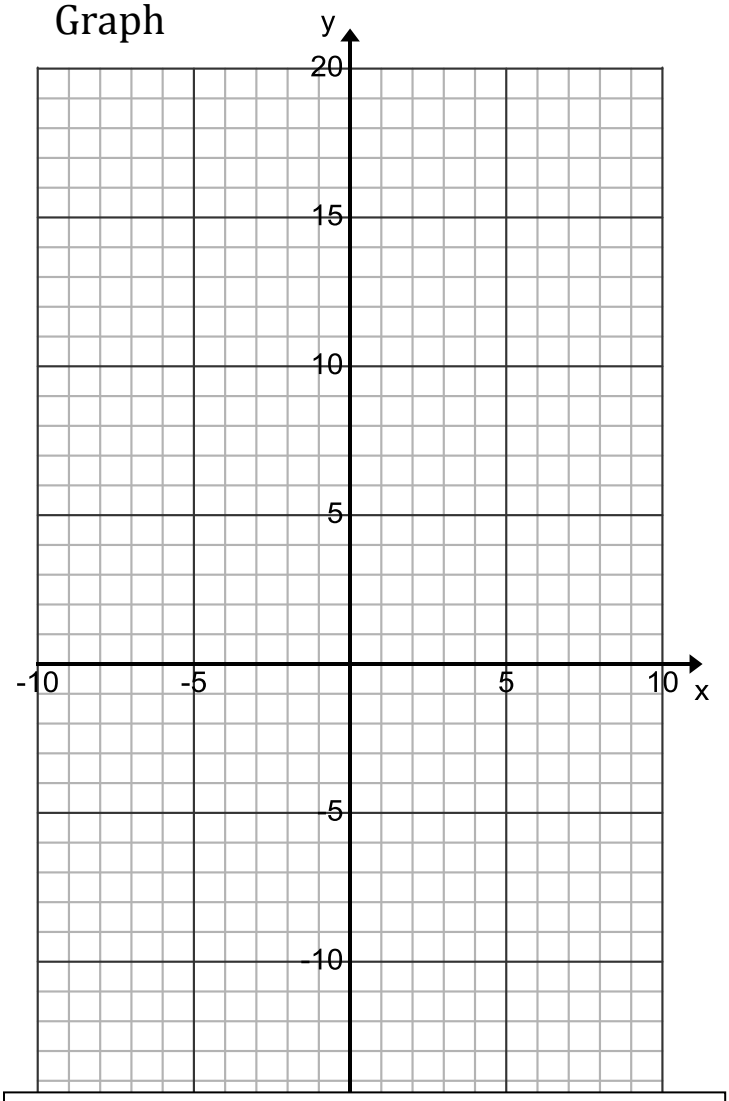
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	0
1	1
2	1.41
3	1.73
4	2
5	2.24
6	2.45
7	2.65
8	2.83
9	3

Graph



Model Equation

$f(x) =$

Situation

Drawing

Record your observations and questions here:

_____

_____

_____

_____

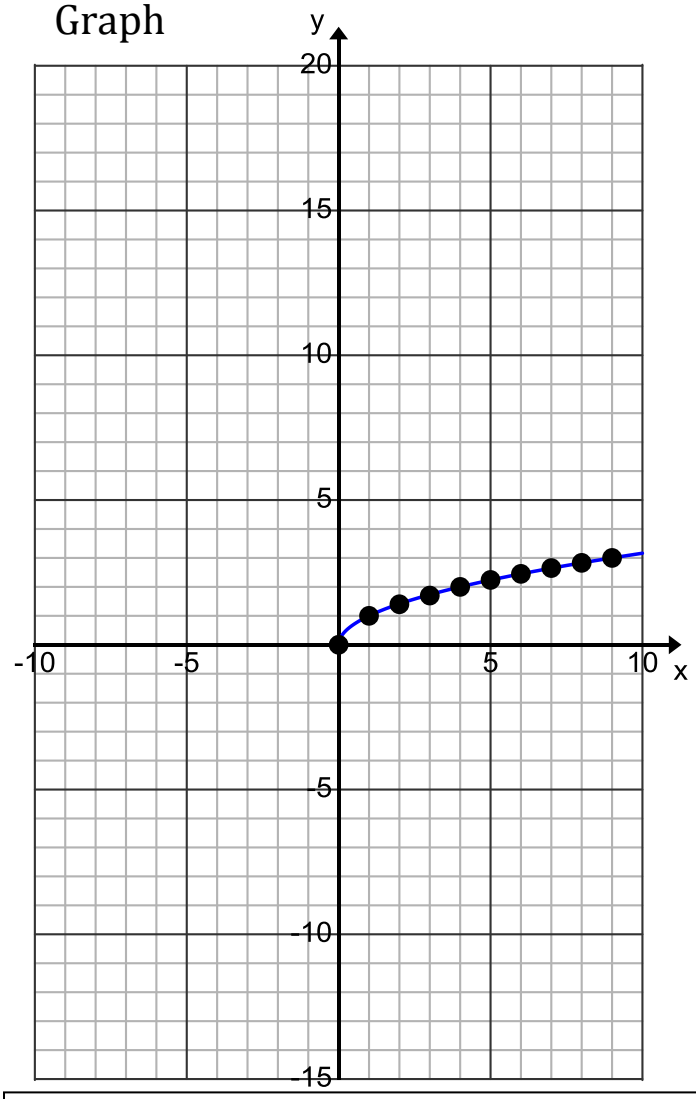
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$f(x) =$

Situation

Drawing

Record your observations and questions here:

_____

_____

_____

_____

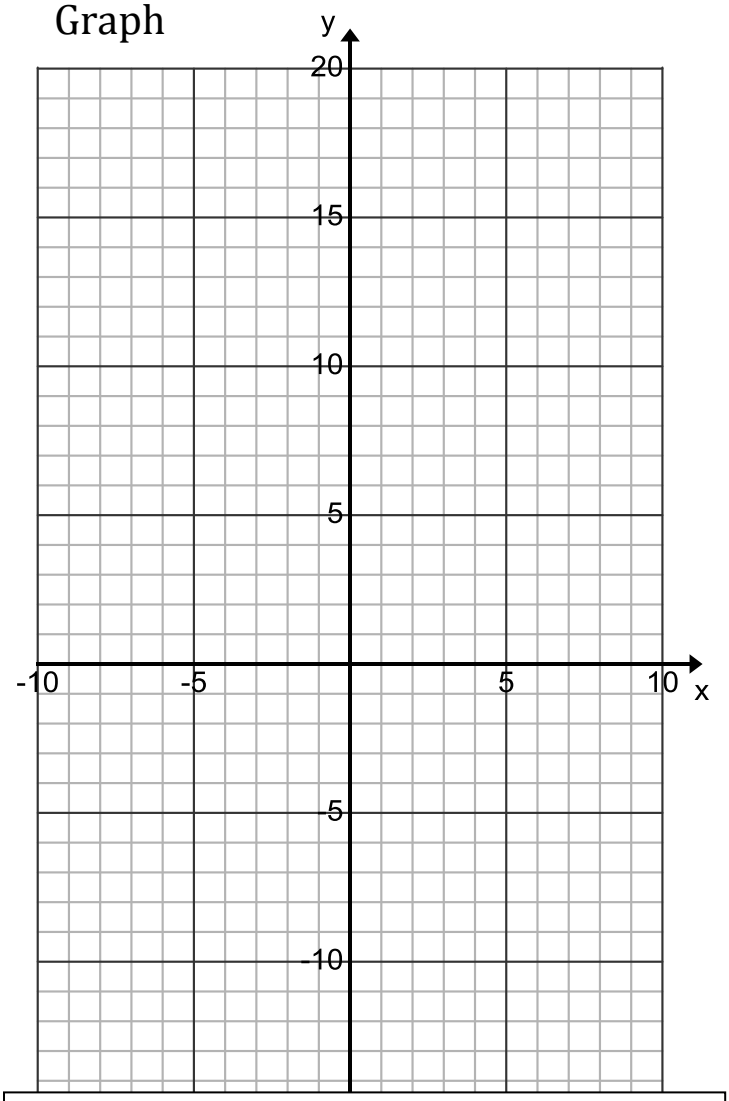
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$f(x) =$

Situation

Simon loves squares. He doesn't have one on the first day, but every day after he makes a square with an area that is one larger than the day before. Then he measures the side of the square.

Drawing

Record your observations and questions here:

_____

_____

_____

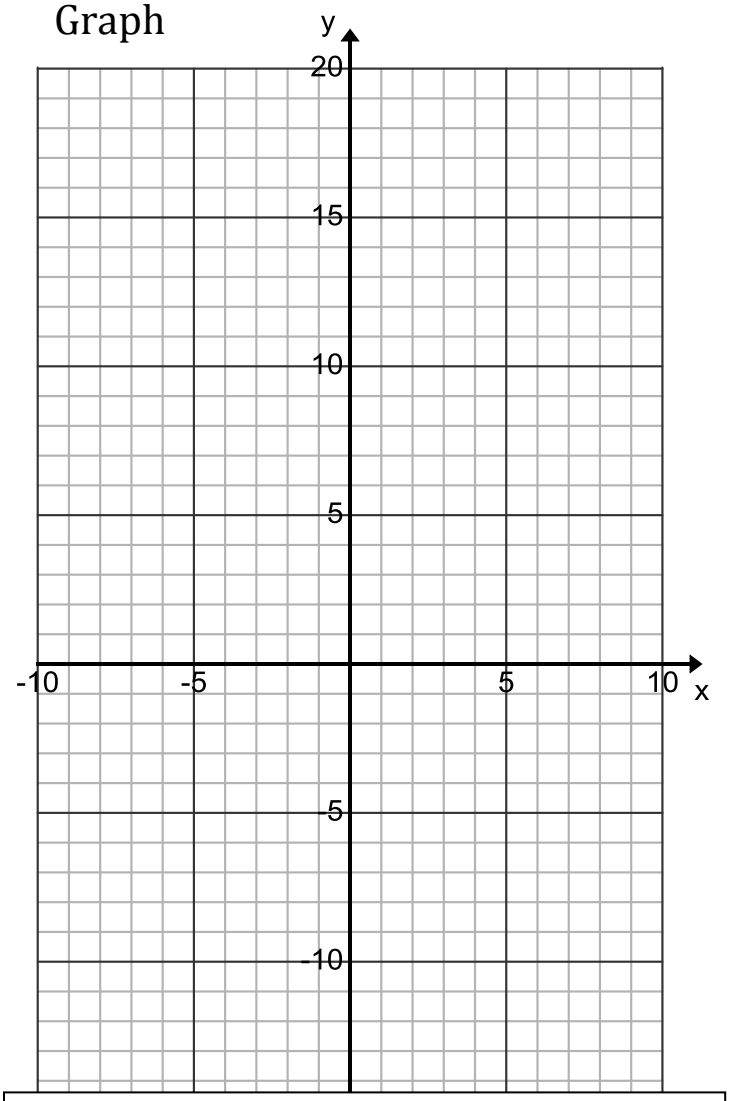
_____

_____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph

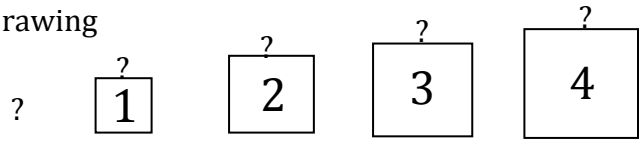


Model Equation

$f(x) =$

Situation

Drawing



Record your observations and questions here:

_____

_____

_____

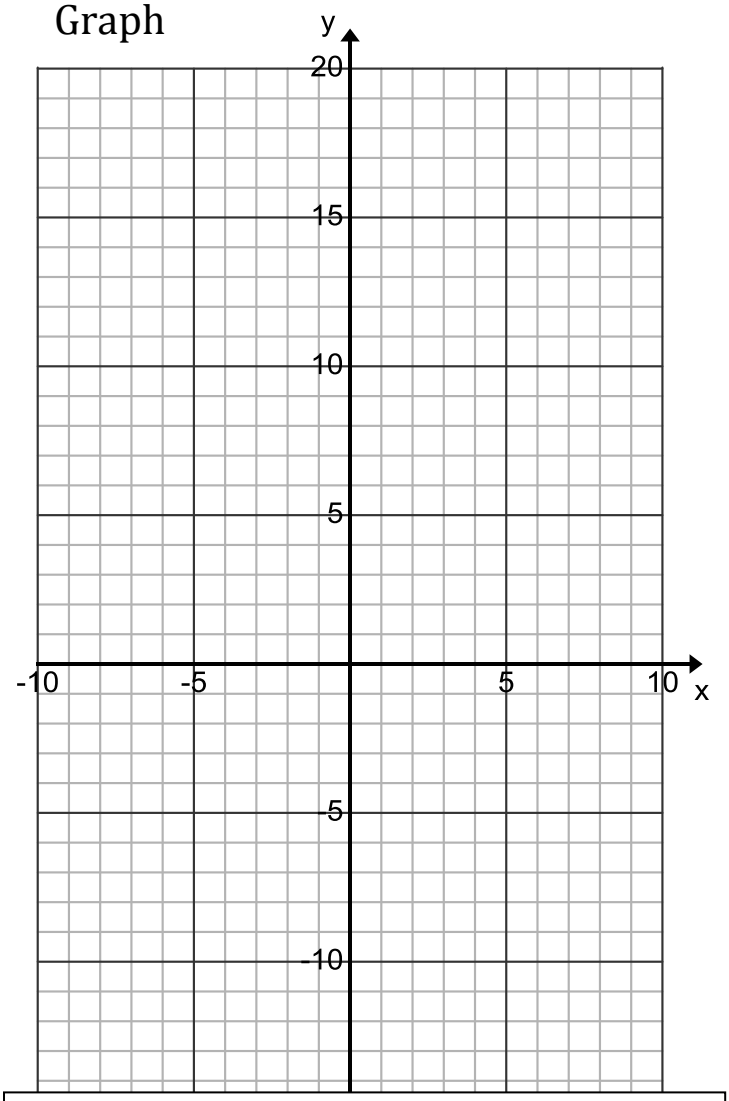
_____

_____

Data Table

x	f(x)
0	0
1	3
2	8
3	15
4	24
5	35
6	48
7	63
8	80
9	99

Graph



Model Equation

$f(x) =$

Situation

Drawing

Record your observations and questions here:

_____

_____

_____

_____

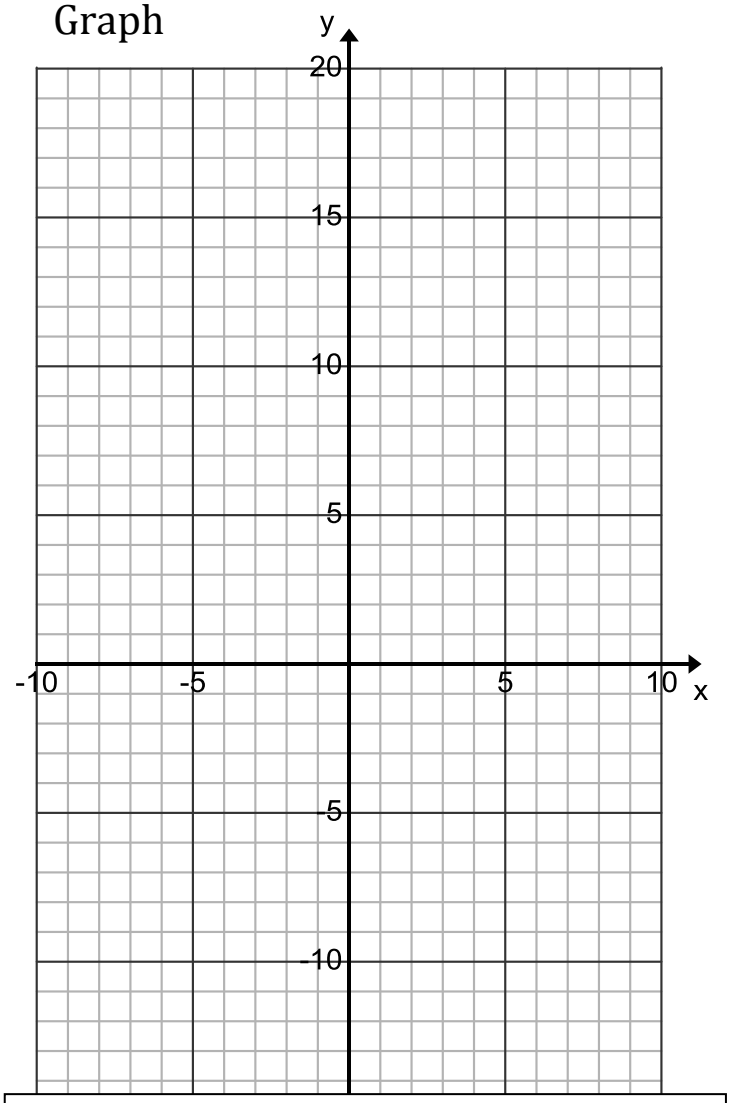
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) = x^2 + 2x$$

Situation

Drawing

Record your observations and questions here:

_____

_____

_____

_____

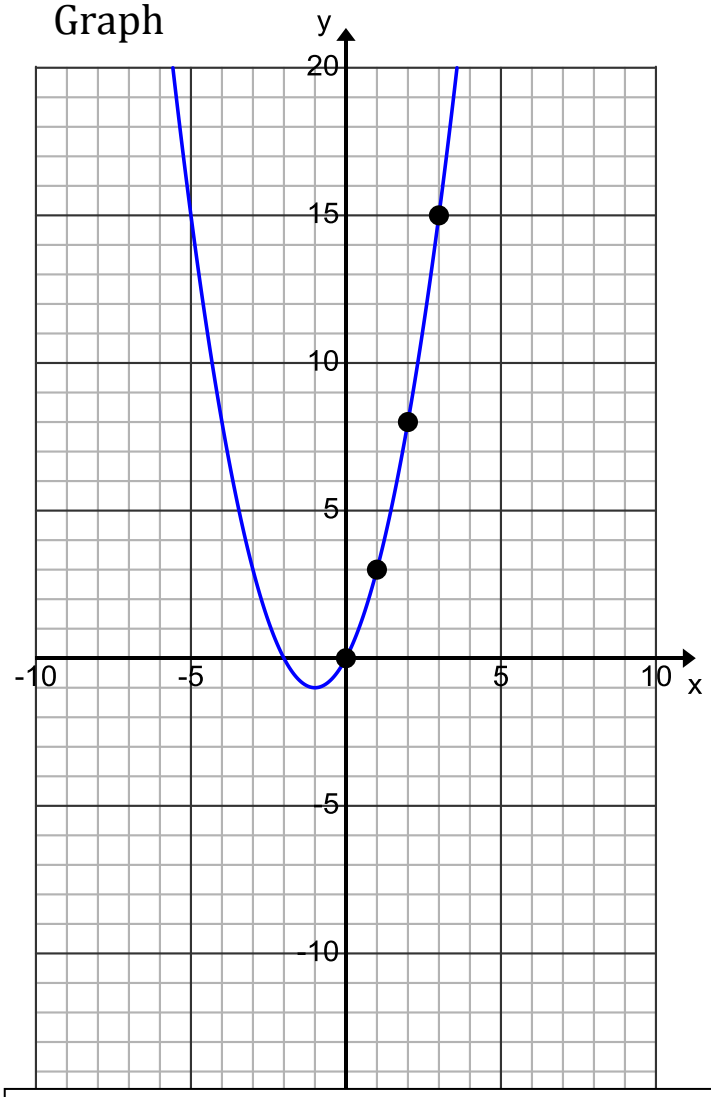
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$f(x) =$

Situation

Drawing

Record your observations and questions here:

_____

_____

_____

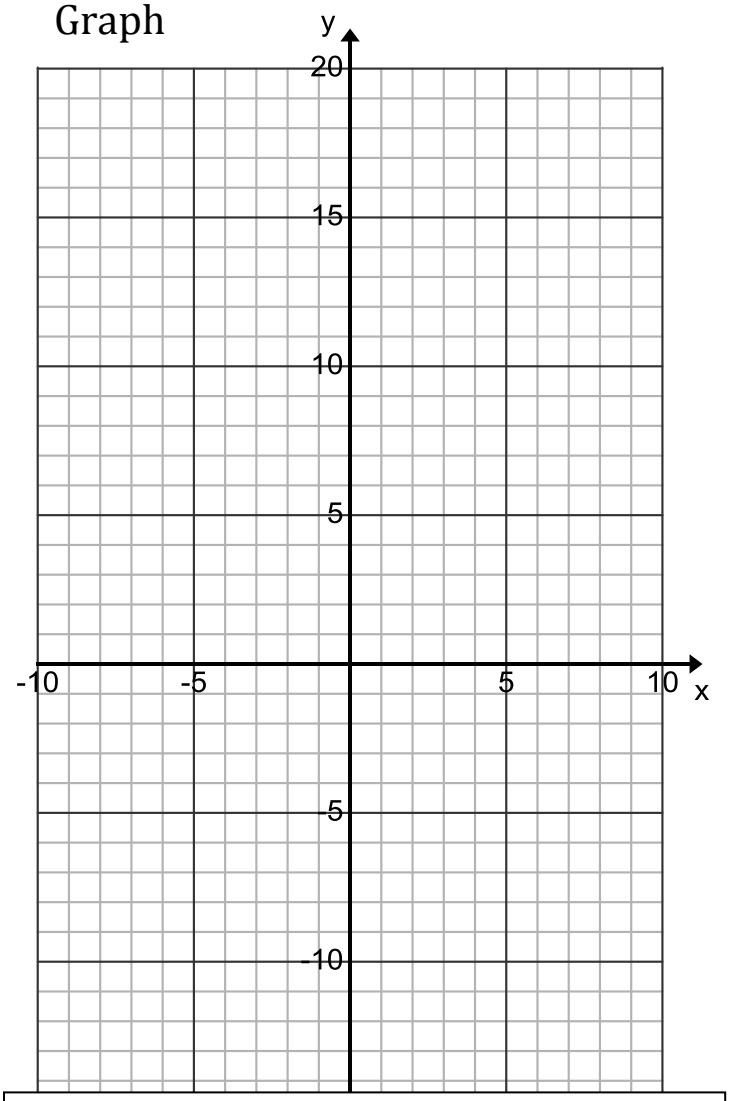
_____

_____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$f(x) =$

Situation

Drawing

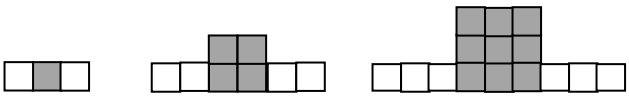


Figure 1 Figure 2 Figure 3

Record your observations and questions here:

_____

_____

_____

_____

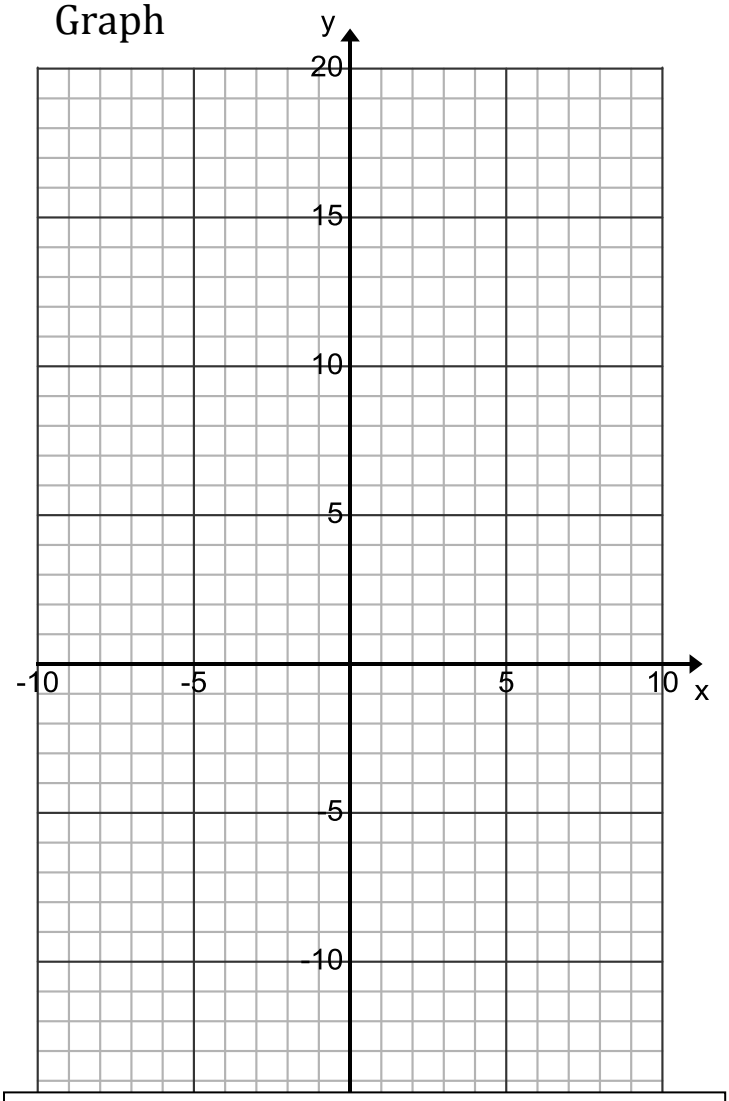
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$f(x) =$

Situation

Bella is building a pattern with square tiles that she calls "Squares with Wings." Each square has two straight wings that are as long as the side of the square.

Drawing

Record your observations and questions here:

_____

_____

_____

_____

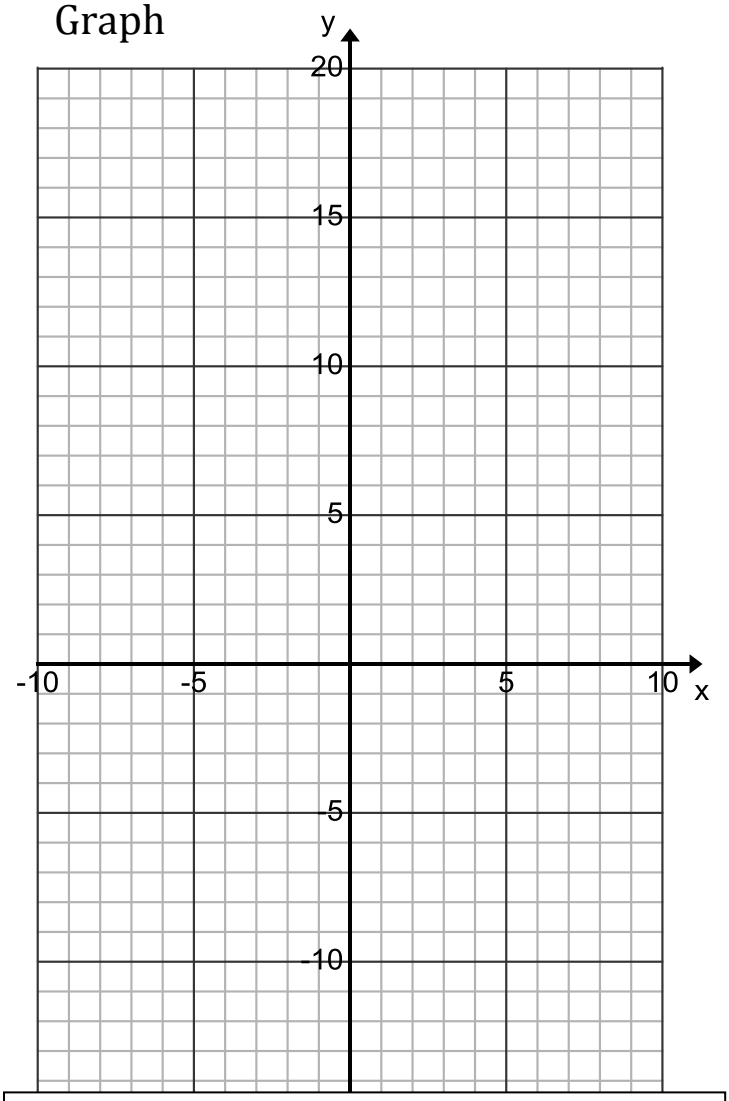
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) = x^3$$

Situation

Drawing

Record your observations and questions here:

_____

_____

_____

_____

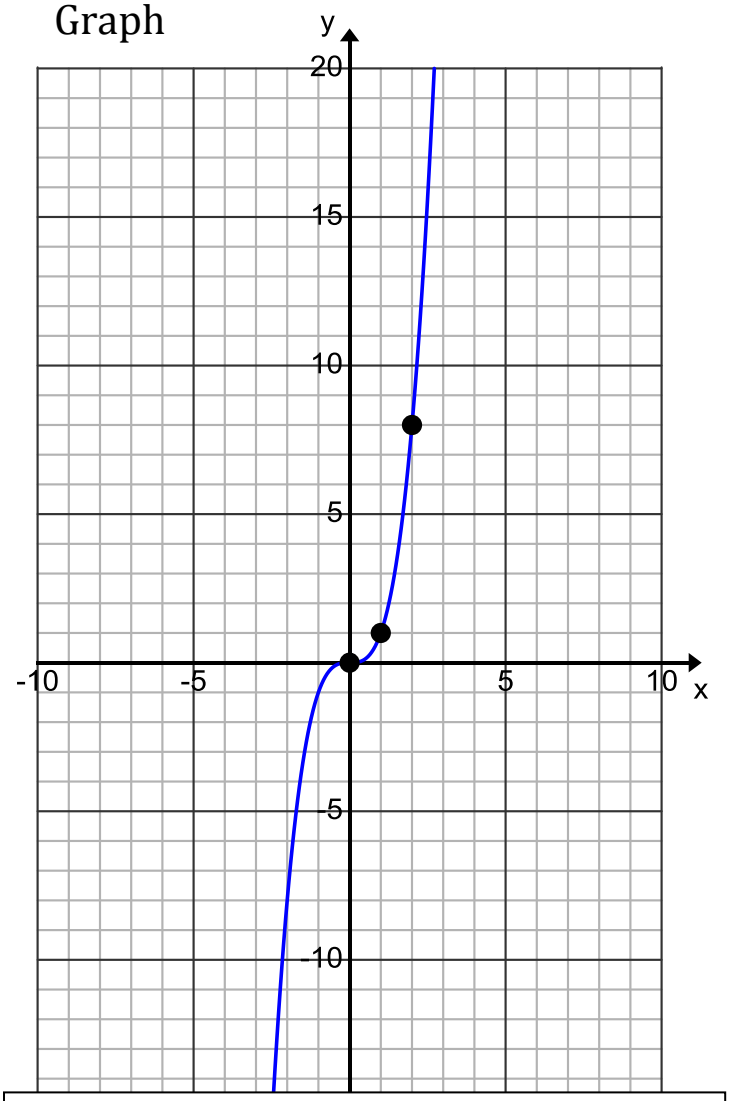
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$f(x) =$

Situation

Drawing

Record your observations and questions here:

_____

_____

_____

_____

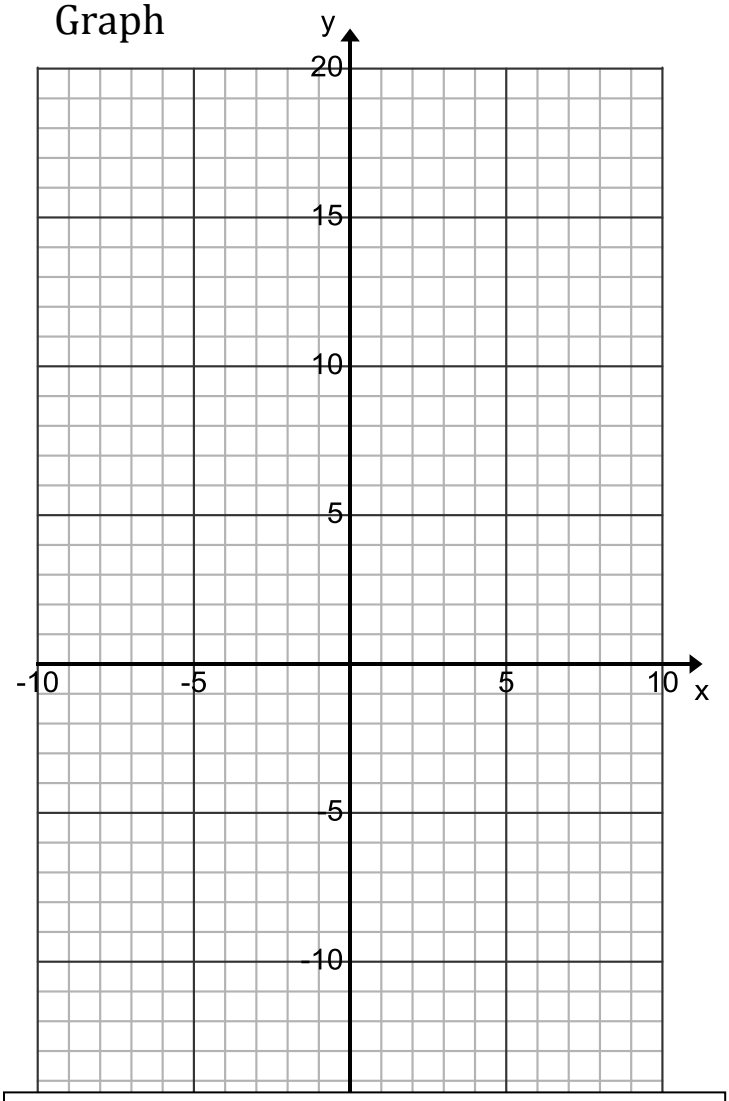
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	0
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729

Graph



Model Equation

$f(x) =$

Situation

Drawing

Record your observations and questions here:

_____

_____

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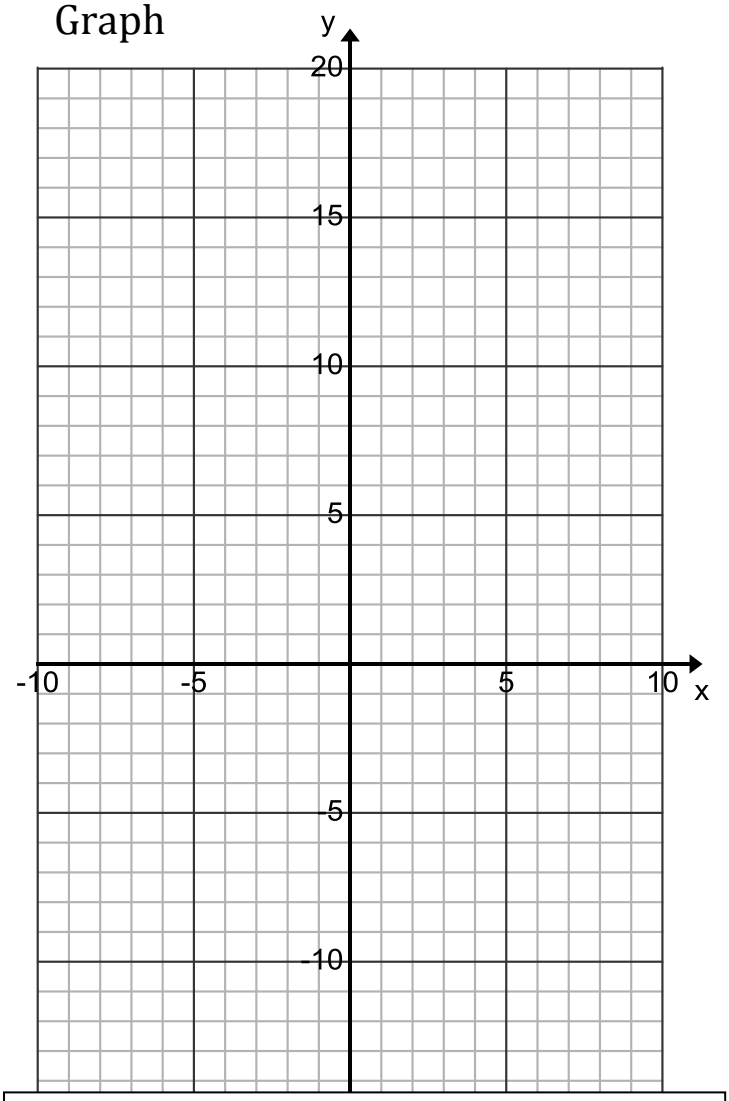
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$f(x) =$

Situation

How many unit cubes does it take to build a larger cube that is "x" long on each edge?

Drawing

Record your observations and questions here:

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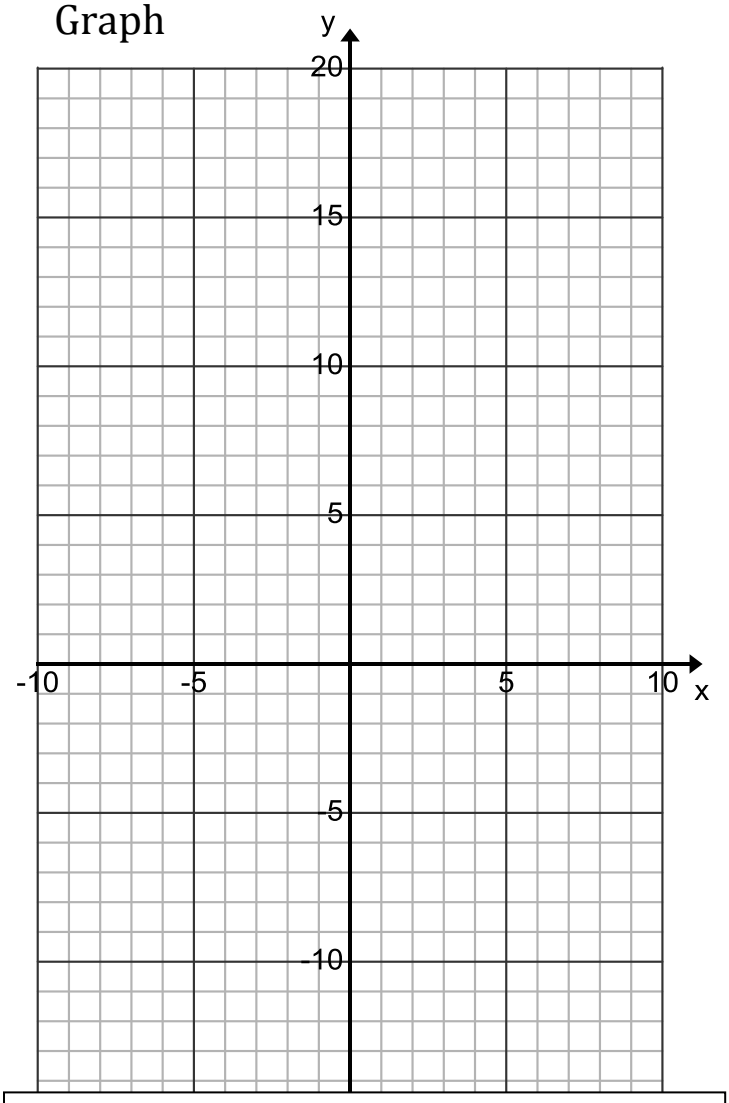
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph

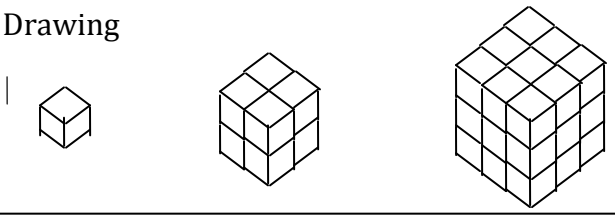


Model Equation

$f(x) =$

Situation

Drawing



Record your observations and questions here:

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_____

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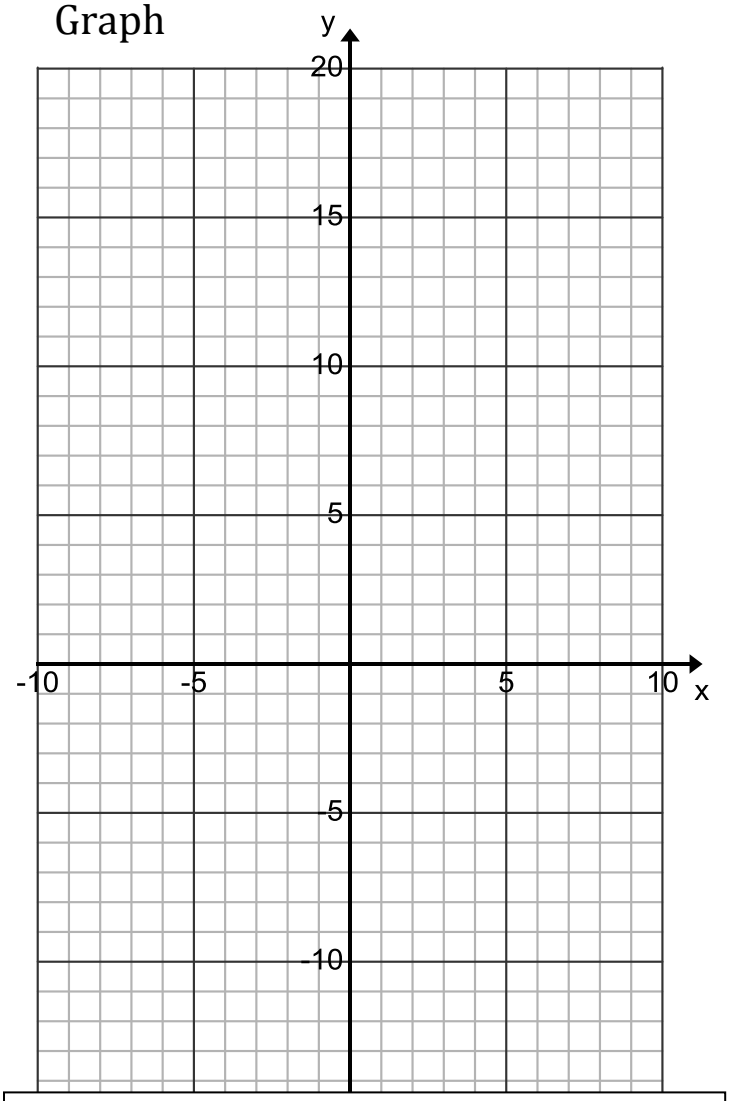
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) = 3x + 1$$

Situation

Drawing

Record your observations and questions here:

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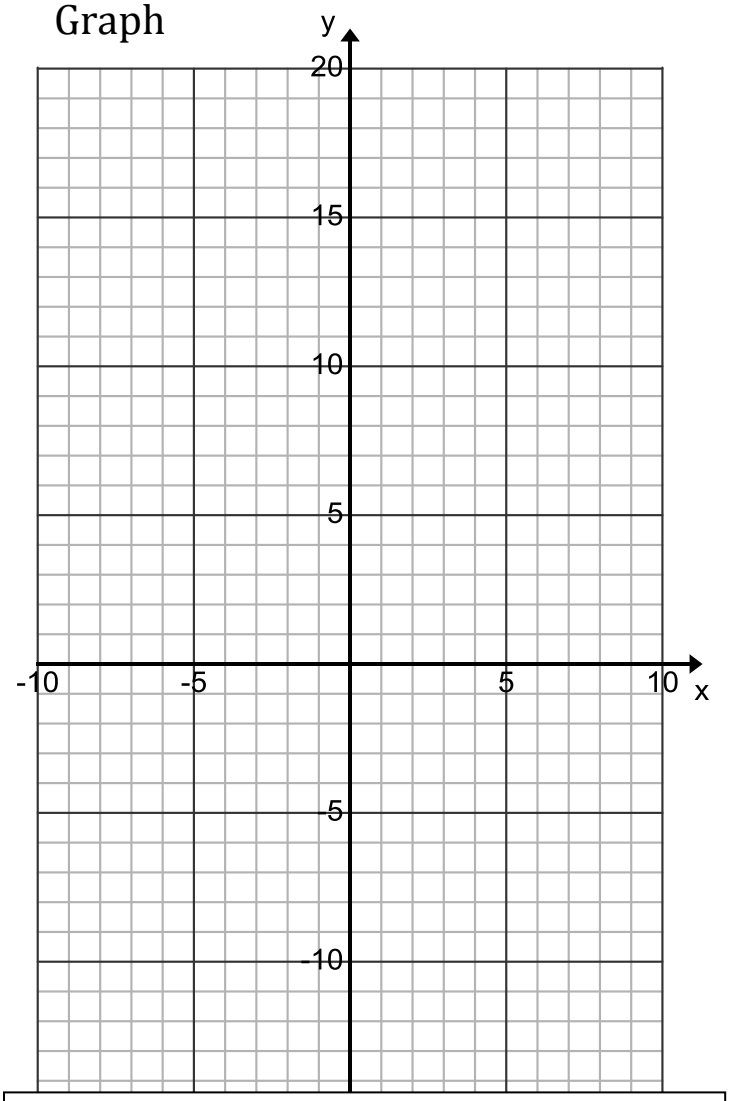
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	1
1	4
2	7
3	10
4	13
5	16
6	19
7	22
8	25
9	28

Graph



Model Equation

$f(x) =$

Situation

Drawing

Record your observations and questions here:

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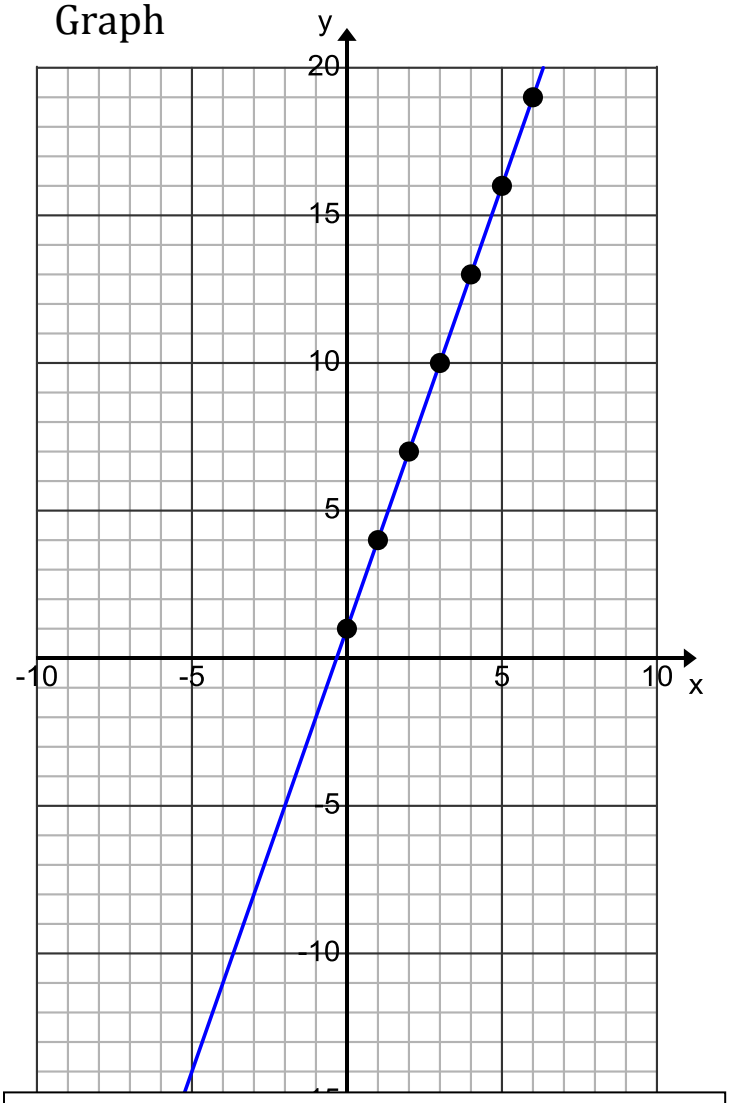
_____

_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$f(x) =$

Situation

Drawing

Record your observations and questions here:

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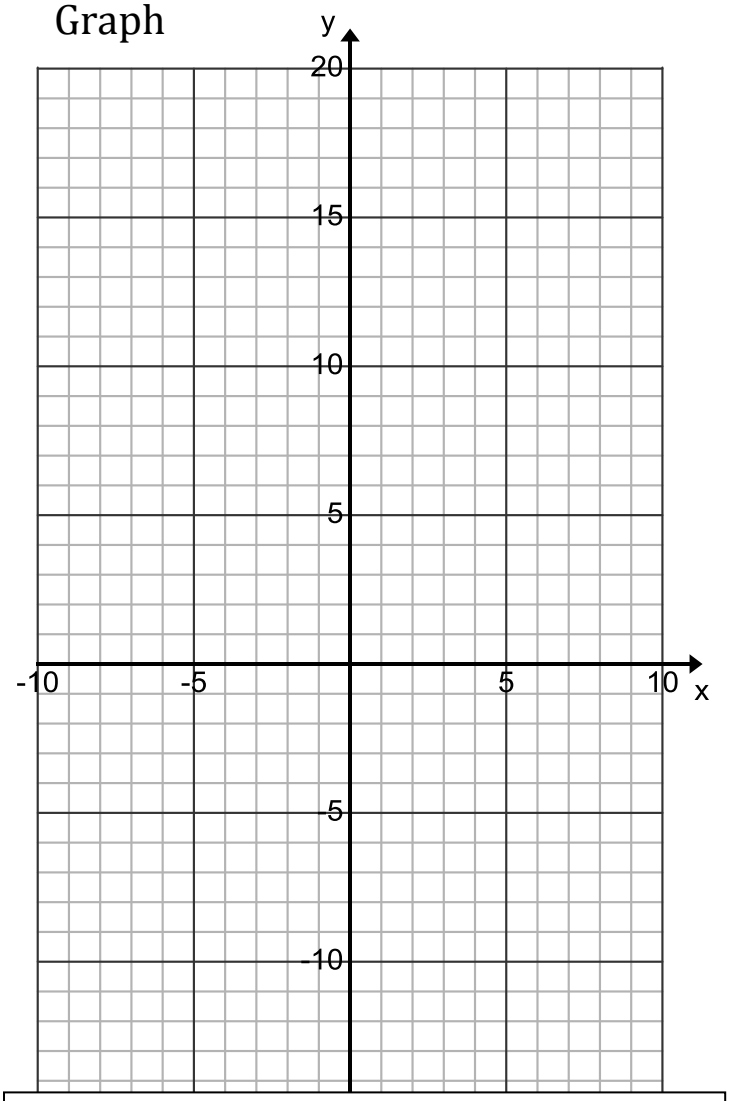
_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$f(x) =$

Situation
Angel is saving money for a new skateboard. He has $1 and knows if he stops buying cookies at lunch he can save $3 each week. How much will he have in x weeks?

Drawing

Record your observations and questions here:

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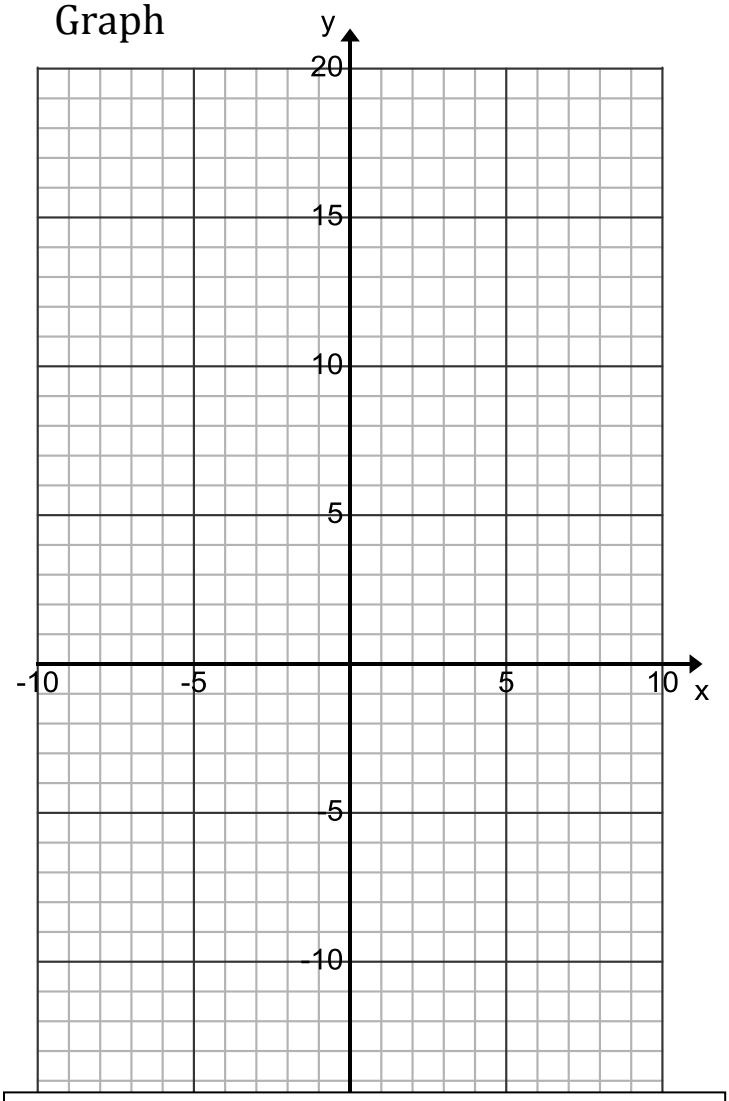
_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$f(x) =$

Situation

Drawing

Record your observations and questions here:

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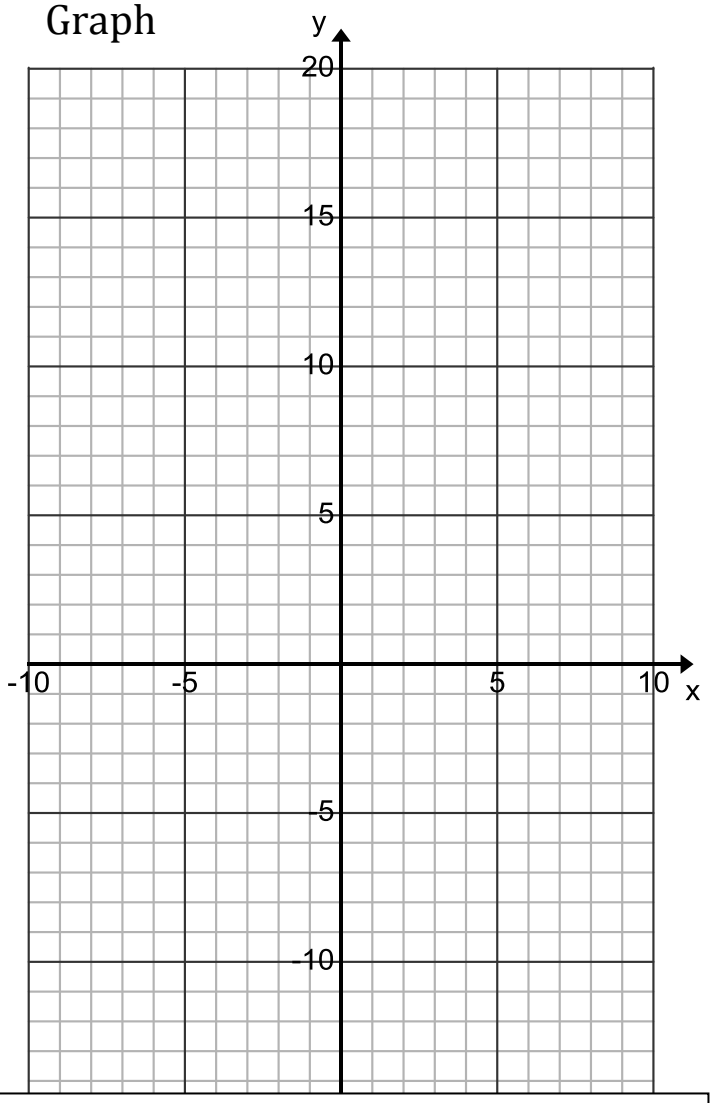
_____ Growth Function Page Name _____

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

Situation

Drawing

Record your observations and questions here:

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Summary of Key Features from Each of the Five Representations

Directions: In each of the boxes below, write several sentences explaining how all of your pages match. Each box is about a different key feature of the function. In your writing, include specific facts from your different representations of the function. Be sure to include observations from all five representations: table, graph, equation, drawing and story.

Y-intercept: How does the y-intercept appear in each representation? What does it mean?

Other Points: Explain how pairs of inputs and outputs are the same in each representation. Explain what a few of them mean.

Rate of Change: How is the function growing? How do you see this growth in each representation? Explain how the rate of change matches across all of the representations.

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SAUSD Common Core Lesson Planner Mathematics Teacher:_____

Unit: A9 **Lesson:** **A9-2-1**	**Grade Level/Course:** CC3/Algebra 2	**Duration: Two periods of 50 Mins Lesson** **Date:**
Common Core and Content Standards	**Understand the concept of a function and use function notation** F-IF 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).	
Materials/ Resources/ Lesson Preparation	Domain and Range Readings from Wikipedia Copies of the following on different colored paper: Three Linear Graphs with Domain/Range Questions Three Quadratic Graphs with Domain/Range Questions Three Cubic Graphs with Domain/Range Questions Three Exponential Graphs with Domain/Range Questions Three Radical (Square Root) Graphs with Domain/Range Questions	
Objectives	**Content:** Students will be able to define the concepts of domain and range of a function. Students will generalize the domain and range of five different function types.	**Language:** Students will use academic vocabulary to explain the ideas of domain and range of a function. Students will listen to and critique other students' conjectures about the domain and range of five function families.
Depth of Knowledge Level	☐ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☐ **Level 3: Strategic Thinking** ☐ **Level 4: Extended Thinking**	
Standards for Mathematical Practice	☐ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☒ **3. Construct viable arguments and critique the reasoning of others.** ☐ **4. Model with mathematics.** ☒ **5. Use appropriate tools strategically** ☒ **6. Attend to precision.** ☒ **7. Look for and make use of structure.** ☒ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	
Academic Vocabulary (Tier II & Tier III) **PROVIDES TEACHER SIMPLE EXPLANATION**	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**
	Function Exponential Linear Quadratic Cubic Radical Square Root Domain Range	Input Output Image

	STUDENTS FIGURE OUT THE MEANING		

Pre-teaching Considerations	(Content) Have students seen/worked with function notation? (Content) Have students seen the graphs of linear, quadratic, cubic, exponential and radical functions? (Pedagogical) Have students moved about the classroom interacting with each other and math content? (Pedagogical) Have students presented conclusions and conjectures to each other before?

Lesson Delivery

Instructional Methods	**Check method(s) used in the lesson:** ☐ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☐ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**
	Prior Knowledge, Context, and Motivation: **Prior Knowledge:** Students have significant experience with linear, quadratic and exponential functions and some familiarity with cubing and square roots. This lesson draws on those experiences and the experience of the previous lesson to further develop the broad concept of functions in general and the specific sub-concepts of domain and range of a function. **Context:** The functions presented in this lesson are without context. **Motivation:** Teamwork and collaboration are used to motivate the finding of meaning in the readings about domain and range as well as the application of that meaning to specific functions.

Lesson Continuum

Body of the Lesson: Activities/ Questioning/ Tasks/ Strategies/ Technology/ Engagement	**Lesson Overview** **Day 1 of 2:** 30 Minutes: • **Mathematical Practice Being Monitored:** Attend to precision. Use a **close reading** strategy to get the meaning of "domain" and "range" from the Wikipedia pages. Be sure to make use of the diagrams and examples within the readings. Possible strategies for working with the Domain reading: • Have students first look at the diagram and the caption below it. Ask them to explain what it means. • Have students read the first paragraph silently, then talk in partners to list things they understood and things they have questions about. • Have students read the second paragraph and explain how it is different from the first paragraph. • Have students look across the page and find the word that is explained as being the opposite of Domain.	**Differentiated Instruction:** **English Learners:** Students who need assistance should be partnered with students who are at a little higher level than they are. **Students Who Need Additional Support:** Students who need assistance should be partnered with students who are at a little higher level than they are.

	Possible strategies for working with the Range reading: • Have students first look at the diagram and the caption below it. Ask them to explain what it means. • Have students look for the two possible ways in which the word "Range" is sometimes defined. • Have students compare and contrast what is said about the functions $f(x) = x^2$ and $f(x) = 2x$. You may also choose to have every other pair work with the Domain reading while the other pairs work with the Range reading. After the pairs have read and discussed their topic, the pairs can split and form new pairs so that a Domain reader pairs with a Range reader. New discussions let each person explain their reading to the new partner. Use a summary strategy to build/record student understanding of the concepts of "domain" and "range." This may include the use of index cards, posters or four corner pages (with a formal definition, an informal description, an example, and a drawing.) *Note: It may seem more efficient to simply give students a simplified and short definition of the words domain and range. This would be faster, but would miss one of the main objectives of this lesson, which is to teach students to find useful information in complex text. If we simplify the text for them, then there is nothing for them to find. Instead, we need to help the students to learn to sort the useful information from the less useful.* 20 Minutes: • **Mathematical Practice Being Monitored:** Construct viable arguments and critique the reasoning of others. The class will be divided into five teams that each get pages with three examples from a family of functions. Ideally, these pages should be color coded with one color for each type of function. Step 1: Each fifth of the class gets one of the types of functions to become experts on. Within the fifth, students should be divided into pairs or trios to make observations and conjectures about the domains and ranges of all functions in this family. Step 2: Pairs and trios within one fifth of the class should discuss with each other their conclusions and conjectures about the domain and range of their family of function. Step 3: Each person must have clear statements written to be ready to share on the following day with classmates who have investigated the other types of functions. **Day 2 of 2:** 10 Minutes: **Mathematical Practice Being Monitored:** Attend to precision. Use a quick-write strategy, possibly with a brief paired discussion, to reinforce and assess both the formal definition and informal understanding of "domain" and "range." 20 Minutes: • **Mathematical Practice Being Monitored:** Attend to precision.	**Accelerated Learners:** If there are extra computers in the classroom the accelerated learners could do their own research on these words.

	Construct viable arguments and critique the reasoning of others. Look for and express regularity in repeated reasoning. Divide the class into teams of 5 so that each team includes one person with each of the five types of functions. Use a timer to give each person 2 minutes to explain their observations and conjectures about the domain and range of their type of function and one extra minute to answer questions. 20 Minutes: • **Mathematical Practice Being Monitored:** Construct viable arguments and critique the reasoning of others. Reason abstractly and quantitatively. Use appropriate tools strategically As a whole class, have a discussion and organize the class findings about the domains and ranges of linear, quadratic, cubic, exponential and radical functions. Use appropriate graphing or calculator technology to support or contradict claims made.	
Lesson Reflection		
Teacher Reflection Evidenced by Student Learning/ Outcomes		

Domain of a function

From Wikipedia, the free encyclopedia

In mathematics, the **domain of definition** or simply the **domain** of a function is the set of "input" or argument values for which the function is defined. That is, the function provides an "output" or value for each member of the domain.[1] The set of values the function may take is termed the range of the function.

For instance, the domain of cosine is the set of all real numbers, while the domain of the square root consists only of numbers greater than or equal to 0 (ignoring complex numbers in both cases). For a function whose domain is a subset of the real numbers, when the function is represented in an *xy* Cartesian coordinate system, the domain is represented on the *x*-axis.

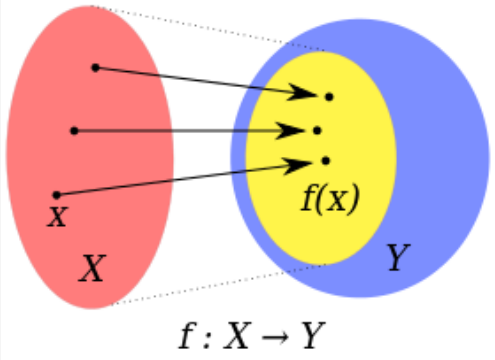


Illustration showing *f*, a function from domain *X* to codomain *Y*. The smaller oval inside *Y* is the image of *f*, sometimes called the range of *f*.

Contents [hide]

Formal definition [edit]

Given a function $f:X\to Y$, the set X is the **domain** of f; the set Y is the codomain of f. In the expression $f(x)$, x is the **argument** and $f(x)$ is the **value**. One can think of an argument as an input to the function, and the value as the output.

The image (sometimes called the range) of f is the set of all values assumed by f for all possible x; this is the set $\{f(x) \mid x \in X\}$. The image of f can be the same set as the codomain or it can be a proper subset of it. It is in general smaller than the codomain; it is the whole codomain if and only if f is a surjective function.

A well-defined function must carry every element of its domain to an element of its codomain. For example, the function f defined by

$$f(x) = 1/x$$

has no value for $f(0)$. Thus, the set of all real numbers, **R**, cannot be its domain. In cases like this, the function is either defined on **R**−{0} or the "gap is plugged" by explicitly defining $f(0)$. If we extend the definition of f to

$$f(x) = \begin{cases} 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

then f is defined for all real numbers, and its domain is $\mathbb{R}$.

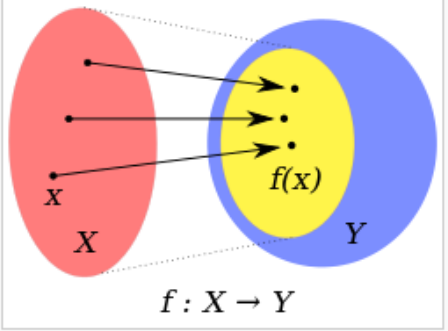
Any function can be restricted to a subset of its domain. The restriction of $g : A \to B$ to S, where $S \subseteq A$, is written $g|_S : S \to B$.

Range (mathematics)

From Wikipedia, the free encyclopedia

This article is about range of a function. For the difference between the largest and smallest numbers in a set, see range (statistics).

In mathematics, the **range** of a function refers to either the codomain or the image of the function, depending upon usage. The codomain is a set containing the function's output, whereas the image is only the part of the codomain where the elements are outputs of the function. For example, the function $f(x) = x^2$ is often described as a function from the real numbers to the real numbers, meaning that the codomain is **R**, but its image is the set of non-negative real numbers. Some books say that range of this function is its codomain, the set of all real numbers, reflecting that the function is real-valued. These books call the actual output of the function the image. This is the current usage for range in computer science. Other books say that the range is the function's image, the set of non-negative real numbers, reflecting that a number can be the output of this function if and only if it is a non-negative real number. In this case, the larger set containing the range is called the codomain.[1] This usage is more common in modern mathematics.



f is a function from domain **X** to codomain **Y**. The smaller oval inside **Y** is the image of f. Sometimes "range" refers to the codomain and sometimes to the image.

Contents [hide]

Examples [edit]

Let *f* be a function on the real numbers $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x$. This function takes as input any real number and outputs a real number two times the input. In this case, the codomain and the image are the same (*i.e.*, the function is a surjection), so the range is unambiguous; it is the set of all real numbers.

In contrast, consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sin(x)$. If the word "range" is used in the first sense given above, we would say the range of *f* is the codomain, all real numbers; but since the output of the sine function is always between -1 and 1, "range" in the second sense would say the range is the image, the closed interval from -1 to 1.

Formal definition [edit]

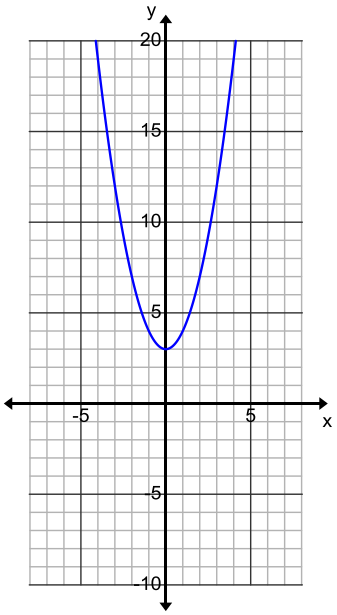
Standard mathematical notation allows a formal definition of range.

In the first sense, the range of a function must be specified; it is often assumed to be the set of all real numbers, and {*y* | there exists an *x* in the domain of *f* such that *y* = *f*(*x*)} is called the image of *f*.

In the second sense, the range of a function *f* is {*y* | there exists an *x* in the domain of *f* such that *y* = *f*(*x*)}. In this case, the codomain of *f* must be specified, but is often assumed to be the set of all real numbers.

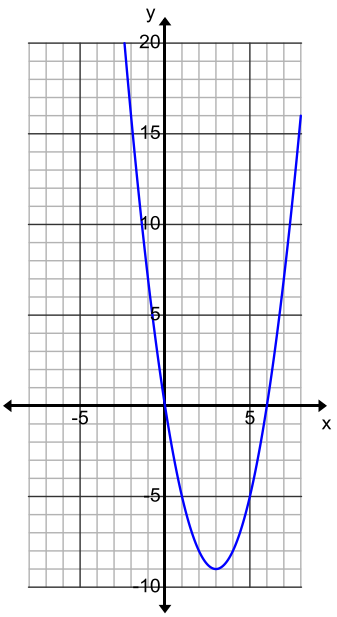
In both cases, image *f* ⊆ range *f* ⊆ codomain *f*, with at least one of the containments being equality.

Three Quadratic Functions



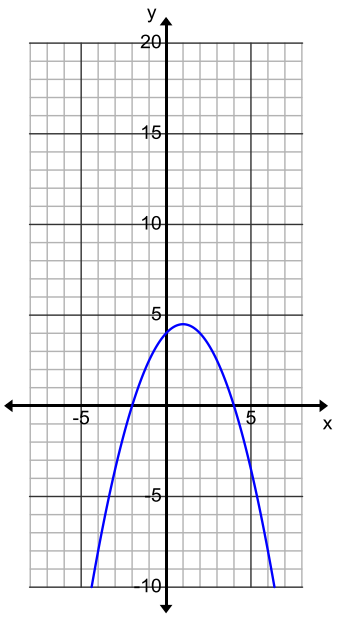
$f(x) = x^2 + 3$

Notes/Observations about the domain and range of f(x):



$g(x) = x^2 - 6x$

Notes/Observations about the domain and range of g(x):

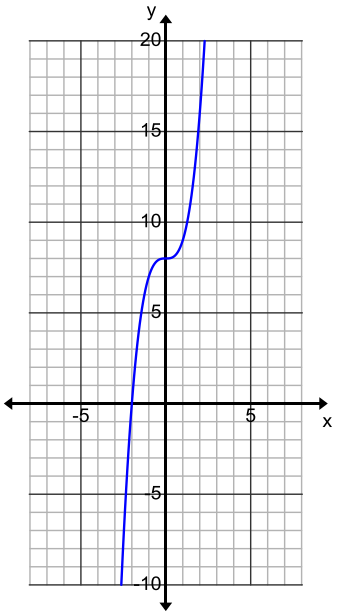


$h(x) = -\frac{1}{2}x^2 + x + 4$

Notes/Observations about the domain and range of h(x):

Observations/Conjectures/Conclusions about the domains and ranges of quadratic functions:
(Please be sure to explain and justify your statements.)

Three Cubic Functions



$f(x) = x^3 + 8$

Notes/Observations about the domain and range of f(x):

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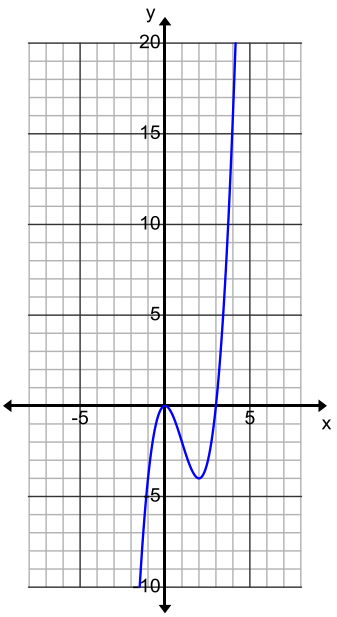

$g(x) = x^3 - 3x^2$

Notes/Observations about the domain and range of g(x):

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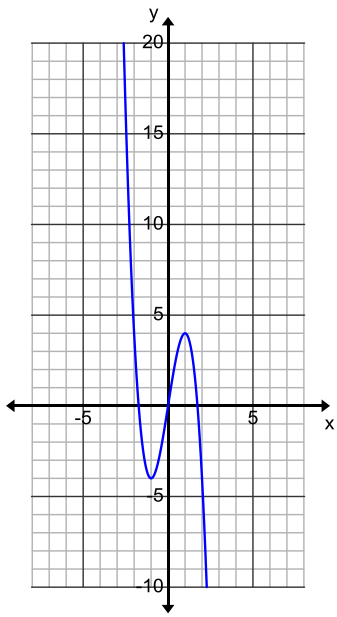

$h(x) = -2x^3 + 6x$

Notes/Observations about the domain and range of h(x):

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Observations/Conjectures/Conclusions about the domains and ranges of cubic functions: (Please be sure to explain and justify your statements.)

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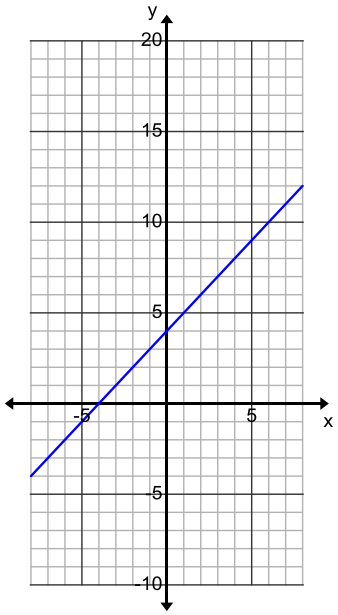
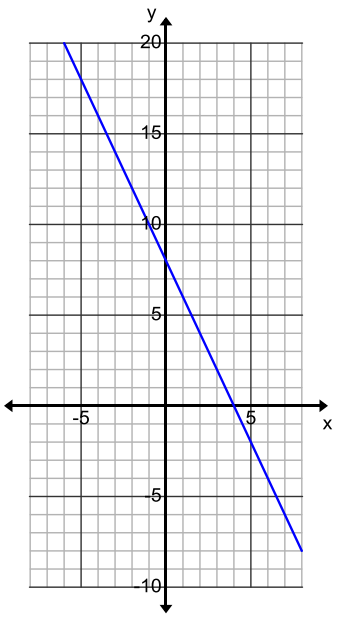
Three Linear Functions



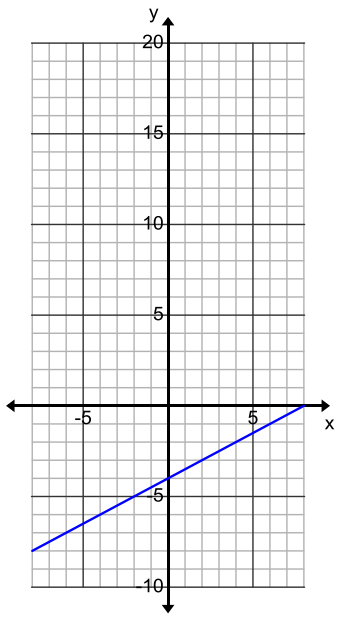
$f(x) = x + 4$

Notes/Observations about the domain and range of f(x):



$g(x) = -2x + 8$

Notes/Observations about the domain and range of g(x):

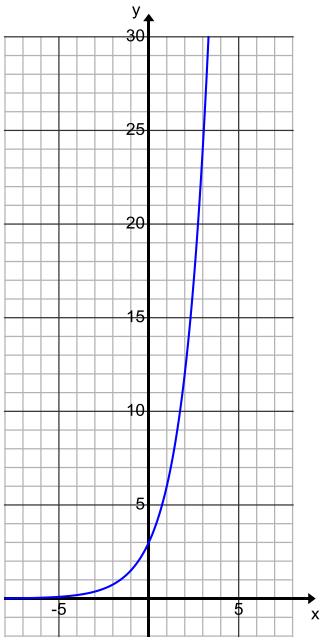


$h(x) = \frac{1}{2}x - 4$

Notes/Observations about the domain and range of h(x):

Observations/Conjectures/Conclusions about the domains and ranges of linear functions: (Please be sure to explain and justify your statements.)

Three Exponential Functions



$f(x) = 3 \cdot 2^x$

Notes/Observations about the domain and range of f(x):

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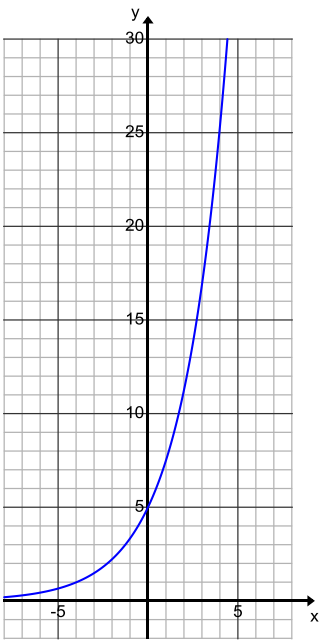

$g(x) = 5 \cdot (1.5)^x$

Notes/Observations about the domain and range of g(x):

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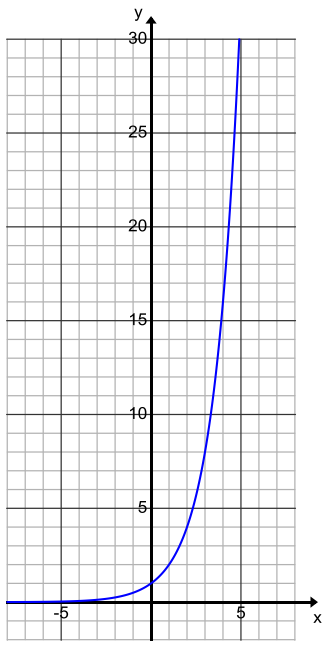

$h(x) = 2^x$

Notes/Observations about the domain and range of h(x):

_____

_____

_____

_____

_____

_____

_____

_____

Observations/Conjectures/Conclusions about the domains and ranges of exponential functions:
(Please be sure to explain and justify your statements.)

_____

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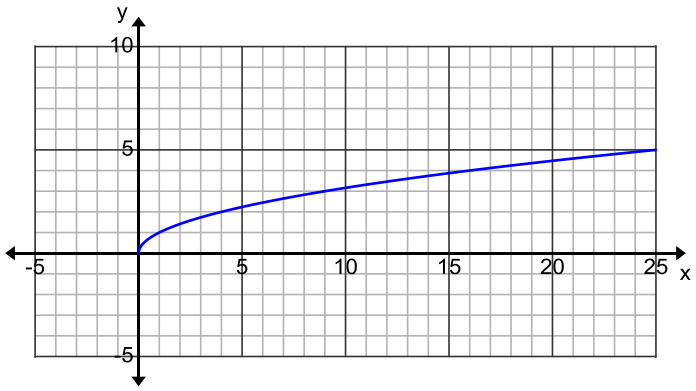
Three Radical (Square Root) Functions



$f(x) = \sqrt{x}$

Notes/Observations about the domain and range of f(x):

_____

_____

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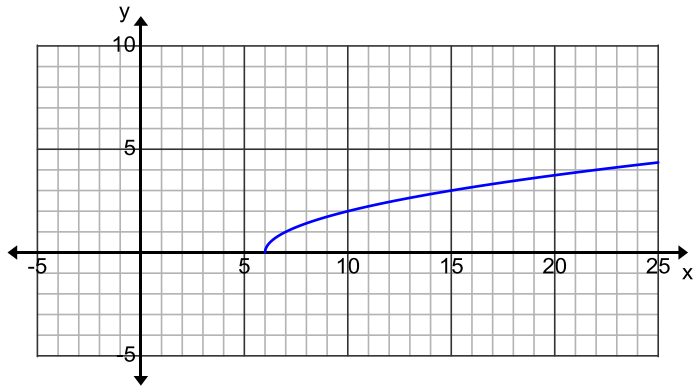


$g(x) = \sqrt{x-6}$

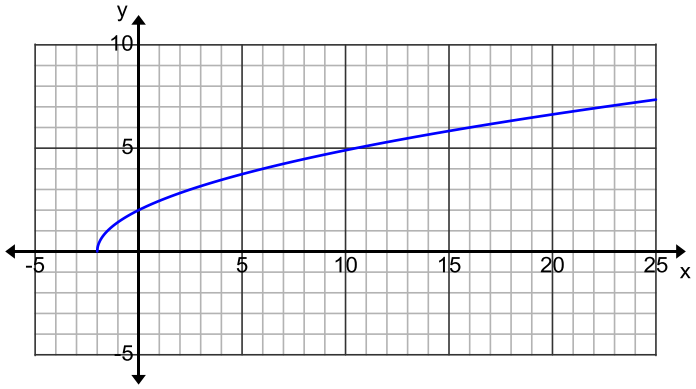
Notes/Observations about the domain and range of g(x):

_____

_____

_____

_____



$h(x) = \sqrt{2x+4}$

Notes/Observations about the domain and range of h(x):

_____

_____

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_____

Observations/Conjectures/Conclusions about the domains and ranges of radical (square root) functions: (Please be sure to explain and justify your statements.)

_____

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SAUSD Common Core Lesson Planner Mathematics Teacher:_____

Unit: A9 **Lesson:** **A9-SA-1**	**Grade Level/Course:** Algebra II / CC3	**Duration: 2 periods of 2 (50 mins) Lesson** **Date:**
Common Core and Content Standards	Interpret the structure of expressions A-SSE.1 – Interpret expressions that represent a quantity in terms of its context. A-REI: Reasoning with Equations and Inequalities Represent and solve equations and inequalities graphically. A-REI.11 F.IF.4 Interpret functions that arise in applications in terms of the context. F.BF.1 – Write a function that describes a relationship between two quantities.*	
Materials/ Resources/ Lesson Preparation	Growth Rate Assessment	
Objectives	**Content:** Students will demonstrate their proficiency in modeling with mathematics by applying conceptual understanding and fluency of Linear and Exponential functions within context.	**Language:** Students will be able to communicate (orally, in writing, and through other representations) about concepts, procedures, strategies, claims, arguments, and other information related to problem solving.
Depth of Knowledge Level	☐**Level 1: Recall** ☒**Level 2: Skill/Concept** ☒ **Level 3: Strategic Thinking** ☒ **Level 4: Extended Thinking**	
Standards for Mathematical Practice	☐ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☒ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☒ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☐ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	
ca bul ary (Ti TEAC HER SIMPL E EXPL	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**

	STUDENTS FIGURE OUT THE MEANING		
Pre-teaching Considerations			

Lesson Delivery		
Instructional Methods	**Check method(s) used in the lesson:** ☒ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☐ **Independent Practice** ☐ **Guided Inquiry** ☒ **Reflection**	
	Prior Knowledge, Context, and Motivation:	
Body of the Lesson: Activities/ Questioning/ Tasks/ Strategies/ Technology/ Engagement	**<u>Lesson Overview</u>** Teacher: Students are to collaboratively work in pairs or groups of four to perform this Performance Task assessment. Students are allowed to use tools strategically. (calculators for computations, but not graphing).	**Differentiated Instruction:** **English Learners:** **Students Who Need Additional Support:** **Accelerated Learners:**
Lesson Reflection		
Teacher Reflection Evidenced by Student Learning/ Outcomes		

Lesson Continuum

Growth Rates

Sisters Courtney and Nina have started an Internet business. Their business started out slow, but business is picking up. They want the business to double how much it earns in the next four months. Courtney and Nina know the company can grow at different rates. They investigate two growth rates, linear and exponential rates. Nina creates the following the following chart for the steady rate.

Linear Rate				
$2,000, _____ ,	_____ ,	_____ ,	_____ ,	$4,000
January	February	March	April	May

1. Complete the sequence of earnings that grows at a linear rate.
   Explain how you know it is linear.

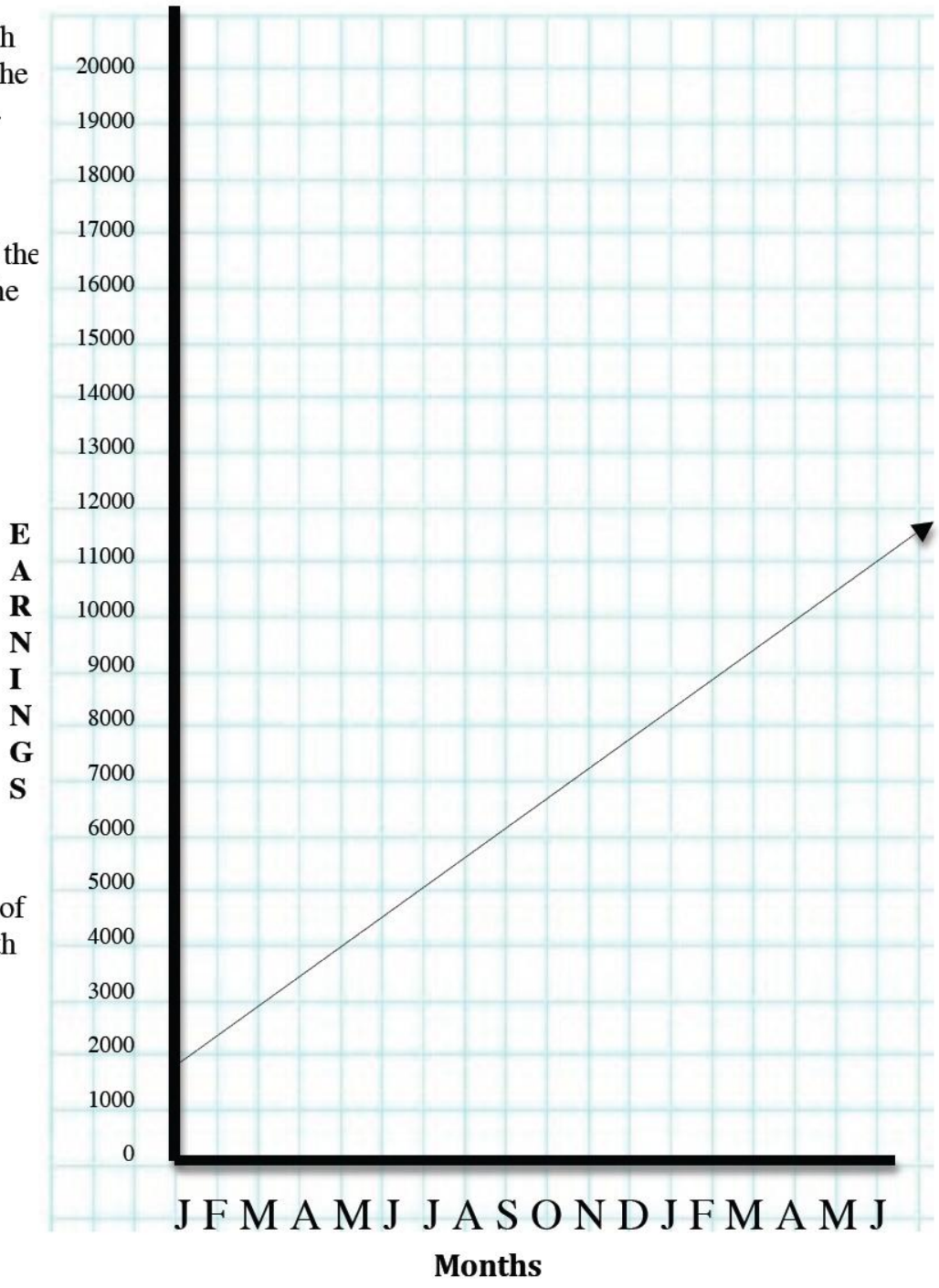
Next the sisters consider the exponential rate. They create a similar chart.

Exponential Rate				
$2,000, _____ ,	_____ ,	_____ ,	_____ ,	$4,000
January	February	March	April	May

2. Complete the sequence of earnings that grow at an exponential rate.
   Show how you determined the earnings for each month.

Courtney drew a graph of the earnings from the linear data calculated.

3. Draw the graph of the exponential data on the same axis.



4. Write an equation of the exponential growth rate.

5. They want to earn a million dollars. If the business grows at that exponential rate, in how money months would that occur? Show how you figured it out.

SAUSD Common Core Lesson Planner Mathematics Teacher:_____

Unit: A9 **Lesson:** **A9-SA-2**	**Grade Level/Course:** Algebra II / CC3	**Duration: 1 period of 50 mins** **Date:**
Common Core and Content Standards	**Analyze functions using different representations** F-IF 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **Interpret functions that arise in applications in terms of the context** F-IF 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **Construct and compare linear, quadratic, and exponential models and solve problems** F-LE 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. F-LE 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
Materials/ Resources/ Lesson Preparation	Functions Assessment page copies	
Objectives	**Content:** Students will match different representations of a function to each other. Students will write equations, make graphs and complete tables given various other representations of a function.	**Language:** Students will use academic vocabulary to explain the type of function used for a set of data. Students will use academic vocabulary to explain domain and range.
Depth of Knowledge Level	☒**Level 1: Recall** ☒**Level 2: Skill/Concept** ☐ **Level 3: Strategic Thinking** ☐ **Level 4: Extended Thinking**	
Standards for Mathematical Practice	☐ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☒ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☒ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☐ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	

Lesson Continuum

Academic Vocabulary (Tier II & Tier III)	PROVIDES TEACHER SIMPLE EXPLANATION	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**
	STUDENTS FIGURE OUT THE MEANING		
Pre-teaching Considerations			

Lesson Delivery		
Instructional Methods	**Check method(s) used in the lesson:** ☐ **Modeling** ☐ **Guided Practice** ☐ **Collaboration** ☒ **Independent Practice** ☐ **Guided Inquiry** ☐ **Reflection**	
	Prior Knowledge, Context, and Motivation:	
Body of the Lesson: Activities/ Questioning/ Tasks/ Strategies/ Technology/ Engagement	**Lesson Overview** This is an end of unit individual assessment of function related skills. This assessment is comprised of questions that require short closed-end answers. This assessment of skills will balance the assessment of problem solving from the MARS performance assessment.	**Differentiated Instruction:** **English Learners:** **Students Who Need Additional Support:** **Accelerated Learners:**
Lesson Reflection		
Teacher Reflection Evidenced by Student Learning/ Outcomes		

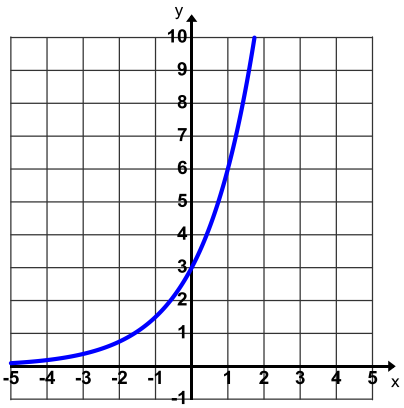
Functions Assessment

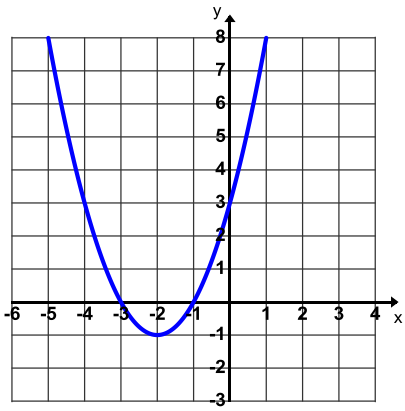
Name _____ Period___

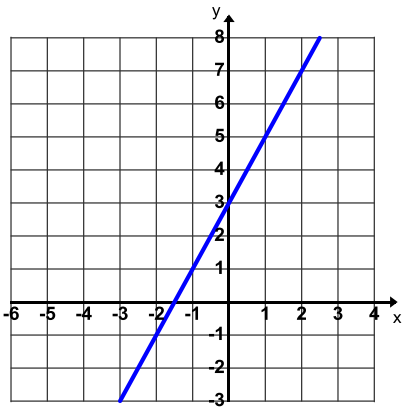
1. Match the equation to the graph.

$f(x) = 2x + 3$ $f(x) = 3 \cdot 2^x$ $f(x) = \sqrt{x} + 3$ $f(x) = 3x + 2$

$f(x) = x^2 + 4x + 3$ $f(x) = 2 \cdot 3^x$ $f(x) = x^3 + 3$







Equation:_____ Equation:_____ Equation:_____

2. Write the equation for the following function.

x	f(x)
0	4
1	9
2	14
3	19
4	24

Equation:_____

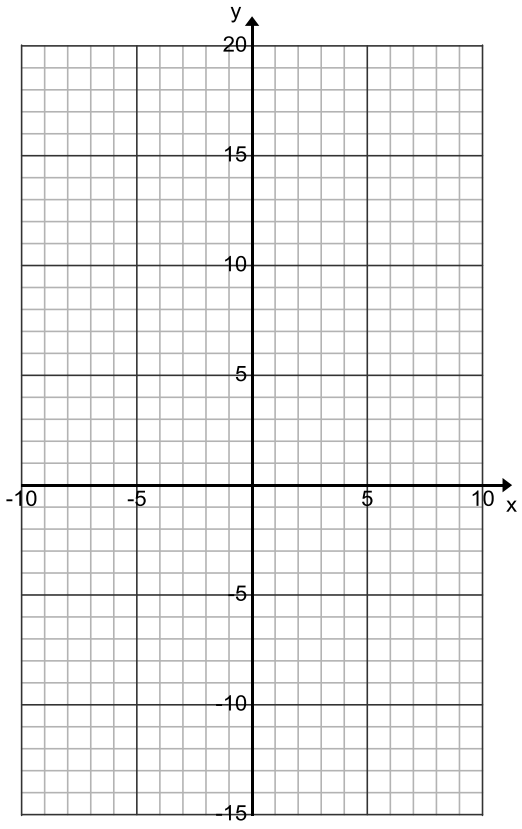
How did you know what kind of equation to use?

_____

_____

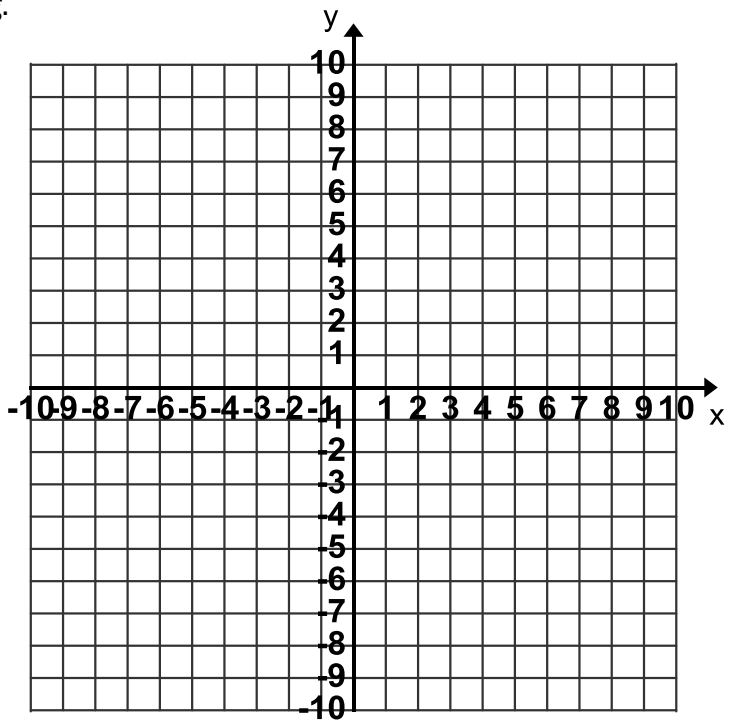
_____

3. Graph the following function: $f(x) = x^3 + 1$



4. Given the function $f(x) = \sqrt{x}$, complete the following.

x	f(x)
0	
1	
4	
9	
16	



What is the domain of this function? Explain what this means.

What is the range of this function? Explain what this means.

5. Write the equation for the following situation: Louis starts with $1 but doubles his money on the first day and continues to double his amount every day.

Equation:_____

How much money will Louis have on Day 5?

When will Louis have more than $1000?

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